Dividend Dynamics and the Term Structure of Dividend Strips

FREDERICO BELO, PIERRE COLLIN-DUFRESNE, and ROBERT S. GOLDSTEIN*

ABSTRACT

Many leading asset pricing models are specified so that the term structure of dividend volatility is either flat or upward sloping. These models predict that the term structures of expected returns and volatilities on dividend strips (i.e., claims to dividends paid over a prespecified interval) are also upward sloping. However, the empirical evidence suggests otherwise. This discrepancy can be reconciled if these models replace their proposed dividend dynamics with processes that generate stationary leverage ratios. Under such policies, shareholders are forced to divest (invest) when leverage is low (high), which shifts risk from long- to short-horizon dividend strips.

RECENT ADVANCES IN CONSUMPTION-based asset pricing models have created an embarrassment of riches in that several competing frameworks are now able to capture many salient features of security prices that were once considered puzzling (e.g., high excess returns and volatilities for stocks, low levels and volatilities for interest rates). Due to these successes, there has been a push to identify additional empirical moments in the hope of distinguishing between the different frameworks. For example, recently there has been considerable interest in the pricing and return properties of so-called dividend strips, which are claims to dividends paid over a prespecified time interval in the future. These assets provide information regarding the term structures of both the

*Belo is with the University of Minnesota and the NBER. Collin-Dufresne is with the Swiss Finance Institute at EPFL and the NBER. Goldstein is with the University of Minnesota and the NBER. This paper was previously entitled “Endogenous Dividend Dynamics and the Term Structure of Dividend Strips.” We thank the Editor, Cam Harvey, as well as the Associate Editor and an anonymous referee for excellent suggestions. We also thank our discussants Lukas Schmid (AFA discussant), Jakub Jurek (WFA Discussant), Ivan Shaliastovich (CAPR and NFI workshop discussant), and Francisco Palomino (Minnesota Macro-Asset Pricing conference discussant) for detailed comments, as well as Ravi Bansal, Jules van Binsbergen, Murray Carlson, Long Chen, John Cochrane, Darrell Duffie, Adlai Fisher, Murray Frank, Dana Kiku, Ralph Koijen, Dimitris Papanikolaou, Ivan Shaliastovich, Jessica Wachter, Jianfeng Yu, and seminar participants at Aalto University/Hanken School of Economics, BI Norwegian Business School, Central Bank of Portugal, Cheung Kong Graduate School of Business, Dartmouth College (Tuck), Hong Kong University, University State Arizona (W.P. Carey), University of Pennsylvania (Wharton), University of Minnesota (Carlson), University of British Columbia (Sauder), University of North Carolina at Chapel Hill (Kenan-Flagler), University of Southern California (Marshall), Vanderbilt University (Owen), American Finance Association 2013, Western Finance Association 2012, Minnesota Macro-Asset Pricing 2012 conference, and the 2012 CAPR/NFI workshop on Time-Varying Expected Returns for many helpful comments. All remaining errors are our own.

DOI: 10.1111/jofi.12242
price and quantity of risk in the economy, and thus may be useful for helping us understand how asset prices are determined in equilibrium. While the return on a portfolio of all dividend strips replicates stock returns, return characteristics of the individual strips provide an out-of-sample test for these competing frameworks.

Indeed, while successful in capturing the asset pricing properties mentioned above, many leading asset pricing models, such as Campbell and Cochrane (CC, 1999), and Bansal and Yaron (BY, 2004), predict that the term structure of expected returns and volatilities on dividend strips are strongly upward sloping. Yet the empirical evidence reported in van Binsbergen, Brandt, and Kojien (BBK, 2012) and van Binsbergen et al. (BHKV, 2013) suggests otherwise. Adding to the puzzle, they also report that short-horizon dividend strips possess low CAPM market betas, high CAPM alphas, and high Sharpe ratios. While Boguth et al. (2011) question some of the findings in these papers, all seem to agree that the strongly upward-sloping term structures predicted by BY and CC are inconsistent with the historical evidence on returns of dividend strips.

Below, however, we demonstrate that the counterfactual implications of these leading asset pricing models can be eliminated without altering their fundamental economic mechanisms. In particular, neither their proposed preferences nor their consumption processes need be changed—hence, their pricing kernels need not be altered. Instead, all that is necessary to make these models consistent with many of the empirical findings of BBK is to replace their proposed dividend dynamics with processes that are both more economically justifiable and more consistent with the empirical properties of dividends that we identify below. In particular, we show that the term structure of dividend volatility (i.e., the quantity of risk) is decreasing with horizon. This evidence is important because, in contrast to the empirical evidence on dividend strips (for which data are available for only a relatively short sample period), the empirical properties of the term structure of dividend volatility can be estimated from more than a century of data. Importantly, we show that our proposed changes to dividend dynamics do not impact the ability of these frameworks to capture the salient properties of stock returns mentioned previously. Intuitively, this property is due to the dividend irrelevance argument of Modigliani and Miller (1961).

In this paper we consider dividend processes that are internally consistent with capital structure policies that generate stationary leverage ratios. In doing so, our model is able to capture two important properties that, at first blush, seem contradictory. First, compared to unleveraged cash flows of a firm (which we refer to as EBIT), dividends are a leveraged cash flow. It is thus not surprising that claims to dividends (i.e., equity) are more volatile and have higher average historical returns than claims to EBIT (i.e., debt plus equity). Second, over long horizons, the dividend-EBIT ratio should be stationary, implying that long-horizon dividends are no more volatile than long-horizon EBIT.

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1 See also Schulz (2013).

2 Here, we are defining quantity of risk as total volatility, and not necessarily as risk that is priced.
This apparent contradiction can be explained by noting that, when a firm rebalances its debt levels over time to maintain a stationary leverage process, shareholders are being forced to divest (invest) when leverage is low (high). Thus, even if investors follow a static strategy of holding a fixed supply of stock, their position is effectively being managed by the capital structure decisions of the firm. Below, we show that these imposed investments/divestments conceal the leveraged nature of dividends in that, even though dividends are correctly interpreted as leveraged cash flows, over the long run EBIT and dividends are equally risky.

Interestingly, most leading asset pricing models ignore either the leveraged nature of dividends or its cointegration with unleveraged cash flows (or both). Moreover, even if they do account for leverage, they do so in a reduced-form way by introducing free parameters that are not directly tied down to observed leverage ratios. For example, for parsimony, CC specify consumption and dividends as i.i.d. with the same drift, and therefore disregard leverage. BY capture leverage by assuming that dividends have greater exposure to shocks to expected growth rates than does consumption. However, their model does not capture stationarity in the dividend-consumption ratio. Abel (1999, 2005) models cash flows to be of the form \( y^\lambda \), where \( \lambda = 0 \) for fixed income securities, \( \lambda = 1 \) for EBIT, and \( \lambda > 1 \) for dividends. Hence, this framework does not capture stationarity in the dividend-EBIT ratio.

To demonstrate the impact of capital structure policies on the properties of dividends in leading asset pricing models, we investigate modified versions of the BY and CC economies. In particular, we replace their proposed dividend processes by first specifying an unleveraged cash flow (i.e., EBIT) process (with the same functional forms as the dividend processes in BY and CC), and then combine it with a dynamic capital structure strategy that produces stationary leverage ratios. These two ingredients generate an endogenously determined dividend process that is internally consistent with the EBIT process. Claims to this dividend process (i.e., equity) have higher expected returns and higher volatilities than claims to EBIT (i.e., equity plus debt), yet this framework generates a stationary dividend-EBIT ratio.

We investigate two different approaches to modeling joint dividend/leverage dynamics. The first approach directly specifies leverage dynamics and then identifies dividend dynamics from an accounting identity that equates the firm’s cash inflows and outflows. Advantages of this framework are its intuitive nature and its tractability, as it provides analytic solutions for dividend strip prices. Unfortunately, it generates counterfactually large correlations between short-horizon dividends and stock returns. As such, we also investigate a second approach where dividend dynamics are directly specified and leverage dynamics are determined from the accounting identity. In addition to matching leverage dynamics and the term structures of dividend strips, this more flexible framework can be calibrated to generate low CAPM betas, high CAPM alphas, and high Sharpe ratios for the short-horizon strips, consistent with BBK and BHKV.
Our main contributions can be summarized as follows. First, our framework predicts that dividend variance ratios (VRs) are a decreasing function of horizon, because dynamic capital structure decisions that generate stationary leverage ratios force shareholders to divest (invest) when leverage is low (high), in turn shifting risk in dividends from long horizons to short horizons. These predictions differ from both CC, where i.i.d. dividend dynamics generate constant VRs, and BY, where long-run risk generates dividend VRs that increase with horizon. We also provide empirical support for this prediction in that VR tests imply long-horizon dividend volatility is significantly lower than short-horizon volatility. That is, the quantity of risk in dividend strips is a decreasing function of horizon.

Second, our framework generates term structures of expected returns and volatilities for dividend strips that are decreasing in horizon, consistent with the empirical findings of BBK, and in contrast to the baseline models of BY and CC. Intuitively, this occurs because the downward-sloping term structure of the quantity of risk dominates the upward-sloping term structure of the price of risk in the baseline BY and CC economies.

Third, while the early literature on “excess volatility” focused on the ratio of equity volatility to short-horizon dividend volatility (e.g., Shiller (1981), LeRoy and Porter (1981)), the fact that managers can (and do—see Chen (2009)) smooth dividends suggests that the more economically relevant property is the ratio of equity volatility to long-horizon dividend volatility (Marsh and Merton (1986), Shiller (1986)). Interestingly, our framework generates stock return volatility that is higher than long-horizon dividend volatility, even if we specify a constant market price of risk. This result is in contrast to the standard Gordon growth model prediction that long-horizon dividend volatility equals stock return volatility, and in stark contrast to the long-run risk model of BY, which predicts that stock returns are less volatile than long-horizon dividends. The intuition for this result is that, since dividends are cointegrated with EBIT, its long-horizon volatility is equal to the long-horizon volatility of (unleveraged) EBIT. In contrast, stock return volatility is pushed up by a “leverage factor” \( \frac{1}{1-L} \). Thus, for an average leverage ratio of 40%, the stock price volatility is about 67% higher than the long-run dividend volatility in a Gordon growth model framework.

Fourth, the economic mechanism in the model implies that the current leverage ratio should have predictive power for the growth rate of dividends. Note that a stationary leverage ratio policy implies that, if leverage is relatively low today, the firm will issue debt to push it back toward its mean. This action creates two reinforcing effects. First, the debt issued allows firms to increase current dividends. Second, it forces equity holders to divest, and hence it reduces future dividends. These two effects imply that leverage ratios should forecast aggregate dividend growth with a positive slope. We confirm this prediction in the data, despite the well-established fact that aggregate dividends are difficult to forecast.

Finally, while we remain agnostic regarding the robustness of some of the empirical findings of BBK and BHKV, who report that short-maturity dividend
strips have high CAPM alphas and high Sharpe ratios, we show that dividend dynamics in both the CC and BY frameworks can be calibrated to match these findings. This calibration emphasizes that firms have considerable flexibility in choosing their dividend policy without materially impacting firm value. Indeed, in reality, not only do firms obtain flexibility in dividend policy through changes in capital structure, but also through investment decisions, equity sales, and equity repurchases.

Related Literature

Our work builds on and combines several strands of the corporate finance and asset pricing literature. Although for simplicity we focus on economies where neither debt policy nor dividend policy impact firm value (Modigliani and Miller (1958, 1961)), it is worth noting that a policy that generates stationary leverage ratios is consistent with firms following an optimal dynamic capital structure policy that maximizes firm value (Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001)). Although empirical tests for leverage stationarity have shown limited success (e.g., Fama and French (2002)), if target ratios change over the business cycle, then the speed of adjustment may be biased (e.g., Titman and Tsyplakov (2007)). Moreover, Graham and Harvey (2001) report that over 80% of firms acknowledge the setting of a target leverage ratio. While Miles and Ezzel (1980) impose constant leverage ratios, we find that such a policy would lead to an excessively volatile dividend process at short horizons. In contrast, a policy of stationary leverage ratios can be calibrated to match observed dividend volatility. That is, the notion of dividend smoothing is inherent when firms are specified to follow a stationary (rather than constant) leverage ratio policy.

Our paper also builds on the consumption-based asset pricing literature by modifying proposed dividend processes to better match the salient properties of dividends and dividends strip returns. As discussed in BBK, even though most of the models in this literature do not attempt to study the pricing of dividend strips, they do provide theoretical predictions about their values. We focus on the predictions of BY and CC given the importance of these papers. Lettau and Wachter (2007, 2011) and Croce, Lettau, and Ludvigson (2009) investigate models that generate a downward-sloping term structure of returns on dividend strips, but their frameworks are substantially different from BY and CC. It is worth noting that Lettau and Wachter (2007, 2011) exogenously specify dividend dynamics such that shocks to dividends and shocks to expected dividends are negatively correlated. In our setting where EBIT is the underlying state variable, this property occurs naturally, because less (more) dividends paid out today imply more (less) dividends available to be paid out in the future.

Our paper is also related to the literature exploring the implications of cointegration restrictions for asset prices. As discussed in Engle and Granger (1987), cointegration implies predictability. This is important, since many researchers have reported that dividends mostly follow a random walk. However, our VR tests for aggregate dividends show that this random walk assumption is not supported by the data. Moreover, we show that leverage (and past dividend growth rates) has predictive power for explaining dividend growth. Models such as Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006) directly model cointegration between consumption and dividends, but their mechanism is through labor share, and not stationary leverage ratios, which generates our results here. Many other papers investigate the asset pricing implications of cointegration between dividends and consumption. A noncomprehensive list includes Bansal, Dittmar, and Lundblad (2001, 2005), Hansen, Heaton, and Li (2008), Bansal, Dittmar, and Kiku (2009), and Bansal, Kiku, and Yaron (2010). Our paper differs because we focus on the cointegration between dividends and EBIT and investigate its implications for dividend strips. Further, we derive the cointegration relationship endogenously through capital structure policies.

Other related papers include Campbell and Shiller (1988), who find that variation in the dividend yield is driven mostly by changes in discount rates. However, others question the power of return predictability (Stambaugh 1999), Campbell and Yogo (2006). Larrain and Yogo (2008) and Boudoukh et al. (2007) argue that discount rates do not need to be so volatile when focusing on the overall cash flows of the firm rather than just dividends. The issue of dividend growth predictability and smoothing is investigated in Chen (2009) and Chen, Da, and Priestley (2012). Our paper adds to this literature by pointing out that long-run variation in dividends is significantly impacted by the capital structure decisions of the firm. Aydemir, Gallmeyer, and Hollifield (2007) investigate how much of the variation in stock volatility can be explained by time-variation in leverage in a habit formation model. Their focus is thus very different from ours.

The rest of the paper is as follows. In Section I we propose a simple two-period binomial model to intuitively demonstrate that imposing stationary leverage ratios shifts risk from long-horizon dividend strips to short-horizon strips. In Section II we document the empirical properties of dividends and leverage to support our economic approach of linking dividends to capital structure decisions. In Section III we investigate a model that captures long-run risk similar to BY but can be made consistent with the empirical findings of BBK and BHKV. We then demonstrate the robustness of our findings by applying it to a model of habit formation similar to CC in Section IV. We conclude in Section V. Proofs are found in the Internet Appendix.

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5 The Internet Appendix is located in the online version of the article at the Journal of Finance Web site.
Dividend Dynamics and the Term Structure of Dividend Strips

I. A Two-Period Binomial Model

In this section we demonstrate within a simple framework the impact that stationary leverage ratios have on the term structure of dividend strips. In particular, we show that imposing stationary leverage ratios tends to increase short-horizon (and decrease long-horizon) dividend volatility relative to a setup in which managers do not maintain stationary leverage ratios.

We investigate a two-period (i.e., three-date) binomial framework with no bankruptcy costs or tax benefits, so we are in a Modigliani-Miller (1958) world where capital structure decisions do not affect firm value. The exogenously specified EBIT process (denoted by $Y$) is given in Figures 1 and 2. The firm liquidates at date 2. We also assume that we are in a complete markets framework and that the Arrow-Debreu price of a security that pays $1 if and only if an up (down) state occurs is $\frac{1}{3} \left( \frac{2}{3} \right)$. Note that this implies that the risk-free rate is zero. Finally, we assume that the probability of an up state is one-half for all states.

We compare two firms. The first firm maintains a constant level of outstanding debt, as in Modigliani and Miller (1958). The second firm maintains a constant leverage ratio, as in Miles and Ezzel (1980). Note that maintaining a
constant leverage ratio is an extreme case of maintaining a stationary leverage ratio. Consider first a firm that has previously issued a bond that pays $25 at date 1, and will cover these cash flows by issuing another one-period bond with face value of $25. Note that in all states of nature, bondholders are paid off in full, so the corporate bond is riskless. Moreover, since the risk-free rate is zero, the bond price is $25 in all states of nature. The dividend paid (denoted by $D$) in any state $\omega_t$ is equal to the sum of EBIT plus the change in the value of the bond position:

$$D(\omega_t) = Y(\omega_t) + [B(\omega_{t+1}) - B(\omega_t)].$$ (1)

The firm’s equity value $V(\omega_t)$ can be calculated in all states by backward induction as $E^Q_t[D(\omega_{t+1}) + V(\omega_{t+1})]$. We can verify that at date 0 the equity value is equal to the enterprise value ($P(0) = E^Q_0[Y_1 + Y_2] = 100$) minus the debt value ($B(0) = 25$), that is, $V(0) = P(0) - B(0) = 75$, consistent with Modigliani–Miller’s theorem. We report these numbers in Figure 1.

Now, let us consider an otherwise identical firm that follows a dynamic capital structure policy that leads to stationary (in fact, constant) leverage ratios. In particular, assume that the firm maintains a 25% leverage ratio. To do so, the firm must change its level of outstanding debt at date 1. For example, if an up state occurs, then, after paying off the old debt, $25, it will issue $35.25 of new debt (which is 25% of the claim to EBIT: $P_\uparrow = $141) and distribute the net difference plus EBIT as a dividend ($D_\uparrow = 35 + 35.25 - 25 = 45.25$). Analogously, if a down state occurs, then, after paying off the old debt, $25, it will issue $10.50 of new debt (which is 25% of the claim to EBIT: $P_\downarrow = $42) and distribute the net difference plus EBIT as a dividend ($D_\downarrow = 20 + 10.50 - 25 = 5.50$). Date 2 dividend payments are determined analogously after noting that there is no new debt issuance at date 2. The relevant security values and cash flows are shown in Figure 2.

For both of these firms, which differ only in their capital structure decisions in period 1, we compute (i) the standard deviations of period 1 and period 2 dividends, (ii) the date 0 prices of dividend strips $V_T(0) = E^Q_0[D_T]$ for $T = (1, 2)$, (iii) their expected returns and variances, and (iv) the expected returns and return variances for the stocks. We report these statistics in Table I.

Comparing the results across rows in Table I confirms our main insight: imposing stationary leverage ratios tends to increase the variance of short-horizon dividends and decrease the variance of long-horizon dividends. In particular, the one-period variance increases from 56 to 395, and the two-period variance decreases from 8,860 to 7,491, as we move from the constant debt model to the constant leverage ratio model. As we show in the theoretical models discussed below, this result is quite general and not specific to the particular example discussed here.

Table I also shows that this shift in risk from long horizons to short horizons due to the stationary leverage ratio policy impacts the properties of dividend strip returns. Specifically, compared to the firm that holds a constant amount
Properties of Dividends and Dividend Strips in the Binomial Model

This table presents calculations of various key statistics corresponding to two different leverage policies represented in Figures 1 and 2.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Definition</th>
<th>Constant Debt Level</th>
<th>Stationary Leverage Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance dividend 1</td>
<td>$\text{Var}_0 [D(1)]$</td>
<td>56</td>
<td>395</td>
</tr>
<tr>
<td>Variance dividend 2</td>
<td>$\text{Var}_0 [D(2)]$</td>
<td>8,860</td>
<td>7,491</td>
</tr>
<tr>
<td>Dividend 1 strip price</td>
<td>$V^1(0) = E^Q[D_1]$</td>
<td>25</td>
<td>18.75</td>
</tr>
<tr>
<td>Dividend 2 strip price</td>
<td>$V^2(0) = E^Q[D_2]$</td>
<td>50</td>
<td>56.25</td>
</tr>
<tr>
<td>Expected return strip 1</td>
<td>$E \left[ \frac{D(1)}{V^1(0)} \right]$</td>
<td>0.1</td>
<td>0.35</td>
</tr>
<tr>
<td>Variance strip 1</td>
<td>$\text{Var} \left[ \frac{D(1)}{V^1(0)} \right]$</td>
<td>0.09</td>
<td>1.12</td>
</tr>
<tr>
<td>Expected return strip 2</td>
<td>$E \left[ \frac{D(2)}{V^2(0)} \right]$</td>
<td>0.33</td>
<td>0.22</td>
</tr>
<tr>
<td>Variance strip 2</td>
<td>$\text{Var} \left[ \frac{D(2)}{V^2(0)} \right]$</td>
<td>0.98</td>
<td>0.44</td>
</tr>
<tr>
<td>Expected stock return</td>
<td>$E \left[ \frac{V(1)+D(1)}{V(0)} \right]$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Variance of stock</td>
<td>$\text{Var} \left[ \frac{V(1)+D(1)}{V(0)} \right]$</td>
<td>0.58</td>
<td>0.58</td>
</tr>
</tbody>
</table>

of debt, the firm that maintains a constant leverage ratio experiences an increase in expected return and variance on the one-period dividend strip and a decrease in the expected return and variance on the two-period dividend strip.

Interestingly, note that the one-period expected return and return variance of the stock at date 0 are not impacted by future capital structure policies, as reported in the last two rows of Table I. This result is consistent with Modigliani–Miller’s dividend irrelevance theorem. Looking ahead, this dividend irrelevance will imply that the change in dividend dynamics due to imposing stationary leverage ratios does not adversely affect the ability of the CC and BY models to capture salient features of stock returns.

II. Empirical Support

In this section, we document several properties of dividends and leverage ratios in the U.S. data. These properties provide empirical support for the key features of the model that drive our results. First, we show that the term structure of dividend volatility (i.e., the quantity of risk) is decreasing with horizon. That is, long-horizon dividends are less risky than both what i.i.d. and long-run risk models would predict (see the theoretical section below). Second, we provide support for the assumption that the aggregate leverage ratio is stationary. In addition, we characterize salient properties of leverage and dividends that will be used to calibrate (and interpret) the modified versions of BY and CC below. We show that short-horizon log-dividend changes are very
weakly correlated with index returns (which is consistent with short-maturity strips having low CAPM betas, as reported in BBK). Finally, we provide empirical support for the economic mechanism linking dividends and capital structure decisions, which, as discussed in the introduction, implies that the aggregate leverage ratio should forecast aggregate dividend growth with a positive slope.

A. Data

The two key variables required for our empirical analysis are aggregate dividends and the aggregate leverage ratio. Our main data source is the Flow of Funds Accounts of the United States (Board of Governors of the Federal Reserve System) Table B.102: Balance Sheet of Nonfarm Nonfinancial Corporate Business, and Table F.102: Nonfarm Nonfinancial Corporate Business. We investigate two alternative definitions of aggregate dividends. The first definition is cash dividends (net dividends, from Table F.102, Line 3). Because dividend strips are claims to cash dividends, the characteristics of this time series are the more relevant for pricing strips. The second definition is total shareholder payouts, which is defined as the sum of cash dividends and net equity repurchases (from Table F.102, Line 39). Because this time series more accurately captures the cash flows to the agent who owns the entire equity claim at all times, the characteristics of this time series are more relevant for calibrating representative agent economies. While repurchase programs were mostly nonexistent in the early part of the sample (Fama and French (2001), Grullon and Michaely (2002)), equity issuances have always been a significant component of net equity payouts. All data are for the nonfarm, nonfinancial corporate sector and are available annually since 1946. We extend the data back to 1930 using the data set constructed in Larrain and Yogo (2008), which is available from Motohiro Yogo’s web page.6

The aggregate leverage ratio is defined as the ratio of the total value of liabilities (from Table B.102, Line 21) to the sum of the total value of liabilities and the total market value of equity (from Table B.102, Line 44) (see also Wright (2004) and Frank and Goyal (2008) for similar constructions). Liabilities are the sum of accounts payable; bonds, notes, and mortgages payable; and other liabilities. As with dividends, we extend the data back to 1930 using the data set constructed in Larrain and Yogo (2008). We construct the market value of equity in the nonfarm, nonfinancial corporate sector by replicating the approach in Larrain and Yogo (2008). (We refer the reader to Larrain and Yogo (2008) for further details on the data construction.)

6 Because we use the Flow of Funds Accounts, the measures of dividends used here include both public and private firms. As such, these measures represent the dividends of all the firms in the economy. To help establish the robustness of the findings, we also report in the Internet Appendix the properties of aggregate dividends of publicly traded firms (using a long time series of aggregate dividends that start in 1873), which are more commonly used in the dividend growth and return forecasting literature. The key properties of dividends that we highlight here (variation of the dividend variance ratios across horizons) are comparable across the two samples.
Although the focus of our analysis is on the properties of aggregate dividends, we also examine the properties of a proxy for aggregate earnings before interest and taxes (EBIT). Following Mead, Moulton, and Petrick (2004), we measure aggregate EBIT as net operating surplus.\textsuperscript{7} The data are from the National Income and Products Account (Table 1.14: Gross Value Added of Domestic Corporate Business and Gross Value Added of Nonfinancial Domestic Corporate Business, Line 25), available through the Bureau of Economic Analysis Web site. We exclude retained earnings (Undistributed profits, Line 31) from net operating surplus to be consistent with the endowment economies that we study below. All nominal quantities are deflated by the consumer price index, which is available from Robert Shiller's web page.

\textbf{B. Properties of Dividends}

If dividends follow a random walk, then the variance of dividend growth increases linearly with the observation interval. That is, for example, the variance of two-year dividend growth will equal twice the variance of one-year dividend growth, implying that the ratio of the two variances per unit of time (VR) equals unity. Following the approach of Lo and MacKinlay (1988), we construct the VR statistic for the two dividend series across horizons from 1 to 20 years, and the corresponding \textit{p}-values for the null hypothesis that dividends follow a random walk (i.e., VR = 1). In specifying the null hypothesis, we consider both the general case in which the shocks to dividends can be heteroskedastic (reported as \textit{p}-value), and the case in which the shocks are i.i.d. (reported as \textit{p}-value IID).

To help in the calibration of the theoretical model proposed below, we also report the dividend volatilities (standard deviation) at each horizon. We define dividend volatility over a given horizon using two approaches.\textsuperscript{8} First, the more standard approach is to specify dividend volatility over horizon \(T\) as

\[ \sigma_{T,1}^2 = \sqrt{\left( \frac{1}{T} \right) \text{Var}_0 \left[ \log \left( \frac{D(T)}{D(0)} \right) \right]} . \]  

\textsuperscript{7} Net operating surplus measures business income after subtracting the costs of compensation of employees, taxes on production and imports (less subsidies), and consumption of fixed capital (economic depreciation) from value added, but before subtracting financing costs (such as net interest) and business transfer payments. Net operating surplus is therefore conceptually similar to the financial accounting concept of EBIT.

\textsuperscript{8} These formulas are not the ones used in the variance ratio test of Lo and MacKinlay (1988), who instead use unbiased estimators of the variance by appropriately adjusting for the degrees of freedom. As a result, the standard deviations for these two alternative measures do not exactly match the implied standard deviations used in the reported VR statistics, but the difference between the two is minimal.
Table II
Variance Ratios

This table reports the variance ratios of two definitions of dividends, and of a proxy for EBIT. \( \sigma_{T,j} \), are the per-year standard deviation of the growth rate of \( j = D \) (dividends) or \( E \) (EBIT), for dividend volatility definition \( i = 1, 2 \). VR is the standard variance ratio using the formula in Lo and MacKinlay (1988), \( p \)-value is the corresponding \( p \)-value of the random walk hypothesis assuming a general structure for the shocks \( (p \text{-value}) \) or assuming i.i.d. shocks \( (p \text{-value IID}) \). The sample is annual from 1930 to 2010.

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>Diff</th>
<th>Diff 1</th>
<th>Diff 2</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>15.52</td>
<td>14.15</td>
<td>9.84</td>
</tr>
<tr>
<td>2</td>
<td>16.20</td>
<td>13.89</td>
<td>9.61</td>
</tr>
<tr>
<td>4</td>
<td>15.82</td>
<td>14.50</td>
<td>10.19</td>
</tr>
<tr>
<td>6</td>
<td>15.75</td>
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<td>15</td>
<td>15.69</td>
<td>14.35</td>
<td>10.08</td>
</tr>
<tr>
<td>20</td>
<td>15.67</td>
<td>14.33</td>
<td>10.06</td>
</tr>
<tr>
<td>1 - 10</td>
<td>15.67</td>
<td>14.33</td>
<td>10.06</td>
</tr>
<tr>
<td>1 - 20</td>
<td>15.67</td>
<td>14.33</td>
<td>10.06</td>
</tr>
</tbody>
</table>

Dividend definition 1: Cash dividends

\[
\sigma_{T,1}^D = \sqrt{\left( \frac{1}{T} \right) \left[ \log E_0 \left[ D^2(T)/D^2(0) \right] - 2 \log E_0 \left[ D(T)/D(0) \right] \right]}.
\]

Dividend definition 2: Cash dividends plus net equity repurchases

\[
\sigma_{T,2}^D = \sqrt{\left( \frac{1}{T} \right) \left[ \log E_0 \left[ D^2(T)/D^2(0) \right] - 2 \log E_0 \left[ D(T)/D(0) \right] \right]}.
\]

We also investigate a second definition that can be solved for analytically in our models: \( \sigma_{D,2}^T \)

\[
\sigma_{D,2}^T = \sqrt{\left( \frac{1}{T} \right) \left[ \log E_0 \left[ D^2(T)/D^2(0) \right] - 2 \log E_0 \left[ D(T)/D(0) \right] \right]}.
\]

The top and middle panels in Table II report, for the two definitions of dividends, the per-year standard deviation of dividend growth across each horizon.

\[ \frac{dD}{D} = g dt + \sigma dz, \]

which in integral form can be expressed as

\[ D(T) = D(0)e^{(g-\sigma^2/2)T + \sigma \sqrt{T}}, \]

both definitions produce the result \( \sigma_{D,1}^T = \sigma_{D,2}^T = \sigma \) for all horizons \( T \).

Note that for the case of log-normal (i.e., i.i.d. random walk) dynamics

\[ \frac{dD}{D} = g dt + \sigma dz, \]
For the two alternative measures of dividend volatility ($\sigma_{D,1}$ and $\sigma_{D,2}$), In addition, the table reports the VR test statistic at each horizon with the corresponding $p$-value. The table shows clearly that dividends do not follow a random walk. For the two alternative dividend measures, the VR test statistic decreases strongly with horizon, that is, the standard deviation of dividend growth is much smaller at long horizons than at short horizons, implying predictability in dividends. Thus, the quantity of risk of dividends is a decreasing function of the horizon.

The ratio between short-horizon (e.g., one-year) and long-horizon (10- or 20-year) standard deviations for dividends is always greater than a factor of two regardless of the measure of dividends used or how dividend volatility is computed. For the first measure of dividends (cash dividends), the VR test statistic (i.i.d. case) rejects the hypothesis that dividends follow a random walk for the 4- to 10-year horizon. In the most general case of the statistic ($p$-value) we can reject the hypothesis that dividends follow a random walk at the 10% significance level for the four- to six-year horizon. For the second definition of dividends (cash dividends plus net equity repurchases), the statistical rejection of the random walk hypothesis is weaker, but the downward slope of the term structure of dividend volatility is even steeper than for the first definition of dividends (which does not include net equity repurchases).

The previous results have implications for the evaluation of leading asset pricing models. The large and positive difference between short- and long-run dividend volatility implies we can reject the random walk assumption of CC in specifying dividend dynamics. Moreover, we can reject even more strongly the long-run risk dividend dynamics posited in BY. As first shown by BBK (and as we confirm below), due to long-run risk, dividend growth volatility increases with horizon, in sharp contrast with the data. The empirical results reported here (using an alternative data set of aggregate dividend data of publicly and privately traded firms) confirm the analysis of Beeler and Campbell (2012), who show, using annual aggregate dividend data for publicly traded firms for the 1930–2008 period, that dividend VRs in the U.S. aggregate stock market decrease with horizon.\(^{10}\)

For completeness, we report additional properties of dividends and EBIT that allow us to interpret the data as well as the fit of the modified BY and CC theoretical models that we discuss below. First, shocks to aggregate dividends appear to be unconditionally uncorrelated with the stock market, even though this weak correlation seems to vary with economic conditions, as captured

\(^{10}\) At a fundamental level, the finding that the dividend volatility decreases with horizon reflects negative serial correlation in the dividend growth series. In the Internet Appendix we show formally that past values of dividend growth help predict future dividend growth with statistically significant negative slopes. Thus, an unusually high value of dividend growth today predicts lower dividend growth. It is this negative autocorrelation that drives the decreasing pattern of dividend volatility across maturities.
This table reports results from the estimation of alternative specifications of the following regression: 

\[ d_{t+1} - d_t = a + (b + c \times dp_t) \times r_{t+1}^e + e_{t+1}, \]

where \( d_{t+1} \) is the log aggregate dividend, \( dp_t \) is the lagged dividend-price ratio on the aggregate stock market (normalized to have mean zero and unit standard deviation), and \( r_{t+1}^e \) is the realized excess return on the aggregate stock market. The table reports the OLS estimate of the relevant slope coefficient, Slope, the corresponding Newey–West corrected \( t \)-statistic, \( [t] \), and the adjusted \( R^2 \). Cash corresponds to aggregate dividends measured as cash dividends, and Cash + NR corresponds to aggregate dividends measured as the sum of cash dividends and net equity repurchases. The sample is annual from 1930 to 2010.

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Cash</th>
<th>Cash + NR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>( r_{t+1}^e )</td>
<td>Slope</td>
<td>0.07</td>
</tr>
<tr>
<td>([t])</td>
<td>( [t] )</td>
<td>1.12</td>
</tr>
<tr>
<td>( dp_t \times r_{t+1}^e )</td>
<td>Slope</td>
<td>-0.11</td>
</tr>
<tr>
<td>([t])</td>
<td>( [t] )</td>
<td>-1.06</td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td>-0.53</td>
</tr>
</tbody>
</table>

by variation in the aggregate dividend-price ratio. To show this, we run the following regression:

\[ d_{t+1} - d_t = a + (b + c \times dp_t) \times r_{t+1}^e + e_{t+1}, \]

where \( d_{t+1} \) is the log aggregate dividend, \( dp_t \) is the lagged dividend-price ratio on the aggregate stock market (normalized to have mean zero and unit standard deviation), and \( r_{t+1}^e \) is the realized excess return on the aggregate stock market (the data for the market factor are from Kenneth French’s web page). The interaction term between the lagged dividend-price ratio and market returns allows us to capture time-variation in the dividend beta. This approach is similar to standard empirical analysis of the conditional CAPM (e.g., Ferson and Harvey (1999)). We run the regression with and without the lagged dividend-price ratio.

Table III reports the regression results using the two alternative measures of dividends. The unconditional covariance of dividends with the stock market is essentially zero, regardless of the dividend definition used (see columns 1 and 3). Because short-horizon dividend growth is expected to be highly correlated with returns on short-horizon (i.e., short-duration) dividend strips, this result anticipates that returns of the short-term strip should have a low correlation with returns on the aggregate stock market (and thus, a low market beta, consistent with the findings in BBK). In addition, the regression points to some variation in the conditional dividend beta, suggesting that it is lower in bad economic times when the aggregate dividend-price ratio is high. This

11 The results are very similar if we use the excess returns on the S&P500.
12 In contrast, we would expect long-horizon (i.e., long-duration) strips to be more impacted by changes in discount rates.
effect, however, is only statistically significant for the definition of dividends that includes net equity repurchases.

Finally, the bottom panel in Table II reports the VRs for our proxy for EBIT. Similar to dividends, the volatility of EBIT decreases with horizon, but not as sharply as the dividend series does (especially for the second definition of dividends as cash dividends plus net equity repurchases). Indeed, short-horizon dividend volatility is much higher than short-horizon EBIT, but long-horizon dividend volatility is converging to long-horizon EBIT volatility. This convergence is consistent with dividends and EBIT being cointegrated, and in turn with the hypothesis that, by maintaining stationary leverage ratios, managers shift dividend risk from long horizons to short horizons.

C. Properties of Aggregate Leverage

Previous studies document that leverage ratios are stationary. At the aggregate level, this result is established in Wright (2004) and Frank and Goyal (2008), among others. At the industry level, as discussed in Collin-Dufresne and Goldstein (2001), leverage ratios have remained within a fairly narrow band even as equity indices have increased 10-fold over the past 30 years. At the firm level, Opler and Titman (1997) provide empirical support for the existence of target leverage ratios within an industry. Further, dynamic models of optimal capital structure by Fischer, Heinkel, and Zechner (1989), and Goldstein, Ju, and Leland (2001) find that firm value is maximized when a firm acts to keep its leverage ratio within a certain band.

Figure 3 shows that our empirical measure of aggregate leverage ratio is stationary, consistent with the previous studies. More formally, a regression of changes in the log aggregate leverage ratio ($\Delta \text{Lev}_{t+1}$) on lagged values of the log aggregate leverage ratio ($\text{Lev}_t$) produces the following results (Newey-West (1987) corrected $t$-statistics in parentheses):

$$
\Delta \text{Lev}_{t+1} = -0.129 - 0.137 \times \text{Lev}_t + e_{t+1},
$$

$$\begin{align*}
R^2 &= 5.90 \\
\sigma(e_{t+1}) &= 0.12
\end{align*}
$$

The negative slope coefficient on the lagged value of leverage implies mean reversion (hence, stationarity) in the aggregate leverage ratio.

Converting these parameters from an annual frequency to a monthly frequency for use in our theoretical work gives

$$
\Delta \text{Lev}_{t+1} = -0.0115 - 0.0122 \times \text{Lev}_t + e_{t+1},
$$

$$\sigma(e_{t+1}) = 0.037.
$$

Finally, the economic mechanism in the model discussed in the introduction implies that the current leverage ratio should have predictive power for the

---

13 Our analysis focuses on aggregate gross leverage. In the Internet Appendix we show that aggregate net leverage (which excludes short-term assets such as cash) exhibits similar stationary behavior. See also Wright (2004) for a similar analysis.

14 Additional studies providing empirical support for the claim that leverage ratios are stationary at the firm level include Flannery and Rangan (2006) and Fama and French (2002).
growth rate of dividends. To examine the relationship between leverage and future dividends, we run standard long-horizon predictive regressions (e.g., Fama and French (1989), Lettau and Ludvigson (2002)). Let \( d_t \) be the log of aggregate dividends. The dependent variable in the predictive regression is the \( T \)-year cumulated log-dividend, in which \( T \) is the forecast horizon ranging from 1 to 20 years. The independent variable is \( \text{Lev}_t \), the current value in the log aggregate leverage ratio. Specifically, we run the following forecasting regression:

\[
d_{t+T} - d_t = a + b \times \text{Lev}_t + e_{t+T}.
\]

For each horizon \( T = 1, \ldots, 20 \), we report the estimated slope associated with leverage, the corresponding \( t \)-statistic, and the regression \( R^2 \). In computing the \( t \)-statistic of the slope coefficient, we use standard errors corrected for autocorrelation per Newey and West (1987) with the lag equal to three years plus the overlapping period, and a GMM correction for heteroskedasticity. The correction for autocorrelation is important here due to the use of overlapping data.

Table IV reports the results of the predictability regressions for the two alternative dividend measures. These results show that leverage forecasts future dividends. Consistent with the model mechanism, the slope coefficients in the predictability regression are all positive after the two-year horizon (at the one-year horizon the slope is negative but statistically insignificant). Thus, periods with high leverage tend to be followed by periods with high dividend growth. The slope coefficients are statistically significant between the 6- and 20-year horizons for cash dividends, and between the 10- and 15-year horizons for...
This table reports results from the following long-horizon predictability regression: 
\[ d_{t+T} - d_t = a + b \times \text{Lev}_t + \epsilon_{t+1} \]
where \( d_t \) is the log aggregate dividend and \( \text{Lev}_t \) is the log aggregate leverage ratio. \( T \) is the forecast horizon in years. The table reports the OLS estimate of the slope coefficient associated with \( \text{Lev}_t \), Slope, the Newey–West corrected \( t \)-statistic, \([t]\), and the adjusted \( R^2 \). The sample is annual from 1930 to 2010.

### Dividend definition 1: Cash dividends

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Forecast horizon (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Lev}_t )</td>
<td>Slope</td>
</tr>
<tr>
<td>([t])</td>
<td>(-0.07)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>(-0.14)</td>
</tr>
</tbody>
</table>

### Dividend definition 2: Cash dividends plus net equity repurchases

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Forecast horizon (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Lev}_t )</td>
<td>Slope</td>
</tr>
<tr>
<td>([t])</td>
<td>(-0.06)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>(-1.00)</td>
</tr>
</tbody>
</table>

dividends that include equity repurchases. The regression \( R^2 \) increases from 2.6% at the 4-year horizon to 30.4% at the 20-year horizon for cash dividends, but the \( R^2 \) is smaller for the definition of dividends that includes net equity repurchases.

### III. Long-Run Risk with Stationary Leverage Ratios

We incorporate stationary leverage ratios in the two-channel long-run risk model of BY. We closely follow the calibration of Bansal, Kiku, and Yaron (BKY, 2012) because it matches several empirical asset pricing moments better than the BY calibration. However, as we demonstrate in the Internet Appendix, the accuracy of the approximation they use to solve the model deteriorates quickly when the persistence parameter \( \rho_\sigma \) is pushed above 0.995, so we limit this parameter to that level.\(^1\) This reduces somewhat the risk premium on the volatility channel, but the calibration still matches many empirical asset pricing moments well.

We first demonstrate that the BY model implies upward-sloping term structures for the excess returns and volatilities of dividend strips. We then modify their dividend dynamics by combining an EBIT process (that has the same structural form as BY’s proposed dividend process) with a dynamic capital structure policy that generates stationary leverage ratios. Dividend dynamics are then obtained endogenously. Our approach of specifying EBIT dynamics and leverage dynamics separately is consistent with the standard methodology in the capital structure literature of assuming separation of investment and

\(^1\) BKY set \( \rho_\sigma = 0.999 \).
capital structure policies. We show that this modified model retains the ability to match excess returns and volatilities of equity, while matching salient features of the returns on dividend strips reported in BBK, and the term structure of dividend volatility reported in the empirical section. Finally, we show how the dividend process can be modified further to match the empirical findings reported in BBK and BHKV that short-horizon dividend strips have low CAPM betas, high CAPM alphas, and high Sharpe ratios. To focus on the economically interesting results, all calculations are relegated to the Internet Appendix.

A. Original BY Framework

BY specify log-consumption ($\Delta c_{t+1}$) and log-dividend ($\Delta d_{t+1}$) dynamics as driven by two persistent variables ($x_t, \sigma_t$):\(^{16}\)

$$
\begin{align*}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \epsilon_{c, t+1} \\
\Delta d_{t+1} &= \mu_d + \rho_d x_t + \nu_c \sigma_t \epsilon_{d, t+1} + \nu_d \sigma_t \epsilon_{d, t+1} \\
x_{t+1} &= \rho \sigma x_t + \nu_c \sigma_t \epsilon_{x, t+1} \\
\sigma_{t+1}^2 &= \sigma^2 + \rho \sigma (\sigma_{t}^2 - \sigma^2) + \nu \epsilon_{\sigma, t+1}.
\end{align*}
$$

(6)

The time interval $\Delta t$ is monthly. We closely follow BKY in setting the parameters to $\mu_c = 0.0015$, $\mu_d = 0.0015$, $\rho_d = 2.5$, $\nu_c = 2.6$, $\nu_d = 4.5$, $\rho_x = 0.98$, $\nu_x = 0.038$, $\sigma = 0.0072$, $\rho_x = 0.995$, and $\nu_x = 0.0000028$.

Preferences are specified to be recursive as in Epstein and Zin (1989), which implies that the logarithm of the one-period (time-$t$ conditional) pricing kernel can be written as

$$
m_{t+1} = \theta \log \delta - \theta \psi \Delta c_{t+1} + (\theta - 1)r_{c, t+1},
$$

(7)

where $r_c$ is the return on the consumption claim, $\gamma$ is relative risk aversion coefficient, $\psi$ is the elasticity of intertemporal substitution, and $\theta = \left(\frac{1 - \gamma}{1 - \frac{1}{\psi}}\right)$.

For tractability, we make two approximations. The first is model-specific, and assumes that the log of the price-consumption ratio is approximately affine in the state variables:

$$
z_t \equiv \log \left(\frac{V_t}{C_t}\right) \approx A_x + A_x x_t + A_\sigma \sigma_t^2.
$$

(8)

The second approximation (the Campbell/Shiller approximation) is mechanical, and approximates the log return $r_c \equiv \log R_c$ on the consumption claim to

---

\(^{16}\)This specification has one drawback: variance is modeled as an AR1 that can become negative. We nevertheless keep this specification since it is very tractable and allows us to follow existing literature for the calibration. In principle, this issue could be avoided by specifying $\sigma_t$ to follow an AR1 process, or by directly modeling variance as a compound autoregressive affine process (a true discrete-time square root process) as proposed by Gourieroux, Monfort, and Polimenis (2007) and Le, Singleton, and Dai (2010).
be linear in the log price-dividend ratio (a similar approximation is made for the log stock return):

\[
r_c \equiv \log \left( \frac{V_{c,t+1} + C_{c,t+1}}{V_{c,t}} \right) \approx \kappa_0 + \kappa_1 z_{c,t+1} - z_t + \Delta c_{c,t+1},
\]

where the constants \(\kappa_1, \kappa_0\) are expressed in terms of \(z = A_0 + A_\sigma \sigma^2\).

To identify the pricing kernel, the six parameters \(\{z, \kappa_0, \kappa_1, A_0, A_x, A_\sigma\}\) are determined from the three equations for \(\{\kappa_0, \kappa_1, z\}\) and from the Euler equation

\[
1 = E_t \left[ e^{m_{t+1} + r_{c,t+1}} \right],
\]

which in this conditionally normal framework can be reexpressed as

\[
0 = E_t \left[ m_{t+1} + r_{c,t+1} \right] + \frac{1}{2} \text{Var}_t \left[ m_{t+1} + r_{c,t+1} \right].
\]

By collecting terms linear in \(x\), linear in \(\sigma^2\), and independent of the state vector, this equation generates three more restrictions, which in turn allows the six parameters to be identified. Moreover, the risk premia coefficients \(\{\lambda_c, \lambda_x, \lambda_\sigma\}\) in the equation

\[
m_{t+1} - E_t \left[ m_{t+1} \right] = -\lambda_c \sigma_t \tilde{e}_{c,t+1} - \lambda_x \sigma_t \tilde{e}_{x,t+1} - \lambda_\sigma \nu \sigma \tilde{e}_{\sigma,t+1}
\]

are also identified.

The risk-free rate at date \(t\) is determined from the pricing kernel via

\[
e^{-r_f(x, \sigma)} = E_t \left[ e^{m_{t+1}} \right] = e^{-r_0 - \frac{1}{2} \kappa_0 - r_\sigma \sigma^2}.
\]

Once the pricing kernel has been identified, the same approach is used to identify the claim to dividends. In particular, we combine the proposed dividend dynamics in equation (6) with the equations

\[
z_{d,t} \equiv \log \left( \frac{V_{d,t}}{D_t} \right) \approx F_0 + F_x x_t + F_\sigma \sigma_t^2
\]

\[
r_d \approx \kappa_{bd} + \kappa_{bd} z_{d,t+1} - z_{d,t} + \Delta d_{t+1}
\]

\[
1 = E_t \left[ e^{m_{t+1} + r_{d,t+1}} \right]
\]

and

\[
\bar{r}_{d,t} \equiv \log E_t \left[ e^{(r_{d,t+1} - r_f,t)} \right]
\]

\[
= \nu_\sigma^2 \kappa_{bd} \lambda_\sigma + (\nu_x \kappa_{bd} \lambda_x + \nu_c \lambda_c) \sigma_t^2\]


and the stock volatility is

\[
\sigma_{d,t} = \sqrt{\log E_t \left[ e^{2(\theta_{d,t+1} - r_{d,t})} \right] - 2 \log E_t \left[ e^{(\theta_{d,t+1} - r_{d,t})} \right]}
= \sqrt{\left(\kappa_d, F, \nu \right)^2 + \sigma_t^2 \left[ \nu^2 + \nu^2_d + \left(\kappa_d, F, \nu \right)^2 \right]}
\] (18)

At the steady-state values \( x = 0, \sigma_t^2 = \bar{\sigma}^2 \), the calibration generates the realistic annual expected excess return of \( \bar{r}_{d,t} = 6.1\% \) and volatility \( \sigma_{d,t} = 15.6\% \).

### A.1 Term Structure of Dividends

For the values \( j = (1, 2) \), it is convenient to define

\[
G_d^{(j,T)}(t, d_t, x_t, \sigma_t) = E_t \left[ e^{j \rho_d} \right]
\] (19)
\[
G_d^{(j,T)}(t - 1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = E_{t-1} \left[ e^{j \rho_d} \right]
\]

Since state vector dynamics are affine, it is well known (e.g., Duffie and Kan (1996)) that the solution is of the form

\[
G_d^{(j,T)}(t, d_t, x_t, \sigma_t) = e^{j \rho_d + F_{0,j}(n) + F_{x,j}(n)x_t + F_{\sigma,j}(n)\sigma_t^2}
\] (20)
\[
G_d^{(j,T)}(t - 1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = e^{j \rho_d + F_{0,j}(n+1) + F_{x,j}(n+1)x_{t-1} + F_{\sigma,j}(n+1)\sigma_{t-1}^2},
\] (21)

where we have defined \( n = (T - t) \) as the number of periods between dates \( t \) and \( T \). Note that, for the solution to be self-consistent at \( (t = T) \), we must have the boundary conditions \( F_{0,j}(0) = 0, F_{x,j}(0) = 0, F_{\sigma,j}(0) = 0 \).

By the law of iterated expectations we have

\[
G_d^{(j,T)}(t - 1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = E_{t-1} \left[ G_d^{(j,T)}(t, d_t, x_t, \sigma_t) \right].
\] (22)

Plugging equations (20) and (21) into equation (22), performing the expectation, and then collecting terms linear in \( x, \sigma_t^2 \) and independent of the state vector, we obtain three recursive equations for \( [F_{x,j}(n), F_{\sigma,j}(n)] \).

Setting \( j = 1 \) we can determine the term structure of dividend expected growth rates over horizon \( n \) via

\[
g_{d,n} = \frac{1}{n} \log \left( E_t \left[ e^{d_{t-n} - d_t} \right] \right)
= \left( \frac{1}{n} \right) \left[ F_{0,1}(n) + F_{x,1}(n)x_t + F_{\sigma,1}(n)\sigma_t^2 \right].
\] (23)

Similarly, using both \( j = 2 \) and \( j = 1 \), we define the term structure of dividend volatilities via

\[
\sigma_{d,n} = \sqrt{\left( \frac{1}{n} \right) \left[ \log E_t \left[ e^{2d_{t-n} - d_t} \right] - 2 \log E_t \left[ e^{d_{t-n} - d_t} \right] \right]}
= \left( \frac{1}{n} \right) \left[ (F_{0,2}(n) - 2F_{1,2}(n)) + (F_{x,2}(n) - 2F_{x,1}(n))\sigma_t^2 \right]^{1/2}.
\] (24)
Figure 4. Term structures in the original BY framework. The top left panel shows the term structure of dividend volatilities (standard deviation), the top right panel shows the dividend strips’ excess returns, the bottom left panel shows the dividend strips’ return volatility, and the bottom right panel shows the dividend strips’ Sharpe ratio in the original BY framework. For comparison, note that the annual expected excess return, return volatility, and Sharpe ratio for the claim to dividends (equity) is $r_d = 6.1\%$, $\sigma_d = 15.6\%$, and $SR_d = 0.39$, respectively. The term structures are evaluated at the long-run values ($x_t = 0, \sigma^2_t = \bar{\sigma}^2$).

We plot the term structure of dividend volatilities at the long-run values ($x_t = 0, \sigma^2_t = \bar{\sigma}^2$) in top left panel of Figure 4. Note that, in contrast to the empirical results reported above, the BY model predicts that dividend VRs are increasing with maturity.

A.2. Dividend Strips

The date $t$ claim to the dividend $e^{d_T}$ paid out at date $T$ is defined as

$$V_d^T(t, d_t, x_t, \sigma_t) = E_t \left[ e^{\left( \sum_{i=1}^{\infty} m_{i,t} \right) + d_T} \right].$$

(25)
Note that from the law of iterated expectations we have

$$V_d^T(t-1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = E_{t-1} \left[ e^{\phi_t} V_d^T(t, d_t, x_t, \sigma_t) \right].$$  \hspace{1cm} (26)$$

Again, since the state vector dynamics are affine, the solution is of the form

$$V_d^T(t, d_t, x_t, \sigma_t) = e^{d_t + F_0(n) + F_x(n)x_t + F_\sigma(n)\sigma_t^2}$$  \hspace{1cm} (27)$$

$$V_d^T(t-1, d_{t-1}, x_{t-1}, \sigma_{t-1}) = e^{d_{t-1} + F_0(n+1) + F_x(n+1)x_{t-1} + F_\sigma(n+1)\sigma_{t-1}^2}.$$  \hspace{1cm} (28)$$

The final conditions are $F_0(0) = 0$, $F_x(0) = 0$, and $F_\sigma(0) = 0$.

Plugging equations (27) and (28) into equation (26), performing the expectation, and then collecting terms independent of the state vector and linear in $(x_t, \sigma_t^2)$, we obtain recursive equations for $(F_x(n), F_0(n), F_\sigma(n))$.

Define the date $t$ one-period gross return on a strip that matures at date $T$ via

$$\bar{R}_{t+1}^T = e^{\phi_{t+1}} = \frac{V_d^T(t+1, d_{t+1}, x_{t+1}, \sigma_{t+1})}{V_d^T(t, d_t, x_t, \sigma_t)}.$$  \hspace{1cm} (29)$$

The log expected excess returns for the dividend strips are

$$\nu(V_d^T(t)) = \log E_t \left[ e^{\phi_{t+1} - r_{f,t}} \right]$$

$$= \nu_x F_x(n-1) \lambda_x + (\nu_x \lambda_x + \nu_x F_x(n-1) \lambda_x) \sigma_t^2.$$

We also define dividend strip volatility as

$$\sigma(V_d^T(t)) = \sqrt{\log E_t \left[ e^{2\phi_{t+1} - r_{f,t}} \right] - 2 \log E_t \left[ e^{\phi_{t+1} - r_{f,t}} \right]}$$

$$= \sqrt{(F_x(n-1) \nu_x)^2 \sigma_t^2 + (F_x(n-1) \nu_x)^2 + \sigma_t^2 (\nu_x^2 + \nu_x^2 + \nu_x^2)}.$$  \hspace{1cm} (31)$$

We plot the term structures of excess returns and return volatilities for dividend strips in the top right and bottom left panels in Figure 4. Note that, analogous to the term structure of dividend volatilities, they are both strongly upward sloping. This result is intuitive in that the term structure of dividend volatilities captures the quantity of risk at different maturities, implying that, if the price of risk is not too strongly decreasing with maturity, this finding for dividend strip returns is to be expected. Note that this prediction is in sharp contrast to the empirical findings of BBK. We therefore search for a way to solve this problem while maintaining the salient properties of the BY paradigm.
B. Modified BY Model

Here we modify the BY framework by specifying unleveraged cash flow dynamics, which we refer to as EBIT dynamics $\Delta y_{t+1}$, instead of specifying dividend dynamics $\Delta d_{t+1}$:

$$
\begin{align*}
\Delta c_{t+1} &= \mu_x + x_t + \sigma_x \tilde{e}_{c,t+1} \\
\Delta y_{t+1} &= \mu_y + \rho_y x_t + v_y \sigma_t \tilde{e}_{c,t+1} + v_y \sigma_t \tilde{e}_{y,t+1} \\
x_{t+1} &= \rho_x x_t + v_x \sigma_t \tilde{e}_{x,t+1} \\
\sigma_t^2 &= \sigma_x^2 + \rho_x \sigma_t \tilde{e}_{x,t+1} \tilde{e}_{x,t+1} + v_x \sigma_t \tilde{e}_{x,t+1}.
\end{align*}
$$

(32)

The parameters for the $(c_t, x_t, \sigma_t^2)$ processes are the same as before. Hence, the pricing kernel remains the same. Compared to the dividend calibration, we set the parameters for the $y_t$ process to reflect their unleveraged nature:

$-\mu_y = 0.0015, \rho_y = 1.6, v_y = 1.5, \nu_y = 3.0$.

We define the claim to EBIT as the enterprise value $V_y(y_t, x_t, \sigma_t)$, where $y_t = \log Y_t$. Since EBIT dynamics have the same functional form as dividend dynamics in the previous section, this implies that if we define the following four functions as analogs to equations (8), (9), (19), and (27)

$$
\begin{align*}
z_{y,t} &= \log \left( \frac{V_{y,t}}{Y_t} \right) \approx U_0 + U_s x_t + U_s \sigma_t^2 \\
r_y &= \kappa_{by} + \kappa_y z_{y,t+1} - z_{y,t} + \Delta y_{t+1} \\
G^{d,y}_y(t, y_t, x_t, \sigma_t) &= e^{i y_t + U_{0,y}(n) + U_{s,y}(n)x_t + U_{s,n}(n) \sigma_t^2} \\
V_y^T(t, y_t, x_t, \sigma_t) &= e^{y_t + U_0(n) + U_s(n)x_t + U_s(n) \sigma_t^2},
\end{align*}
$$

then we find the relations

$$
\begin{align*}
\bar{r}_{y,t} &= \sqrt{v_o \kappa_y U_o \lambda_o + (v_x \kappa_y U_s \lambda_s + v_y \lambda_y) \sigma_t^2} \\
\sigma_{y,t} &= \sqrt{(v_y \lambda_y U_y v_y)^2 + \sigma_t^2 \left[ v_y^2 + v_y^2 + (\kappa_y U_y v_y)^2 \right]} \\
g_{y,n} &= \left( \frac{1}{n} \right) \left[ U_{0,y}(n) + U_{s,y}(n)x_t + U_{s,n}(n) \sigma_t^2 \right] \\
\sigma_{y,n} &= \left\{ \left( \frac{1}{n} \right) \left[ (U_{0,y}(n) - 2U_{0,y}(n)) + (U_{s,y}(n) - 2U_{s,y}(n)) \sigma_t^2 \right] \right\}^{1/2} \\
\bar{F}(V_y^T(t)) &= v_o U_s(n - 1) \lambda_o + (v_y \lambda_y + v_x U_s(n - 1) \lambda_s) \sigma_t^2
\end{align*}
$$

(33, 34, 35, 36, 37, 38, 39, 40, 41)
Figure 5. Term structures of EBIT in the modified BY framework. The top left panel shows the term structure of EBIT volatilities (standard deviation), the top right panel shows the EBIT strips’ excess returns, the bottom left panel shows the EBIT strips’ return volatility, and the bottom right panel shows EBIT strips’ Sharpe ratios in the modified BY framework. For comparison, note that the annual expected excess return, return volatility, and Sharpe ratio for the claim to EBIT is $r_{y,t} = 3.44\%$, $\sigma_{y,t} = 9.71\%$, and $SR_{y,t} = 0.35$, respectively. The term structures are evaluated at the long-run values ($x_t = 0$, $\sigma^2_t = \sigma^2$).

\[
\sigma(V_T^y(t)) = \sqrt{\left(U_s(n-1)v_y\right)^2 \sigma^2_i + \left(U_s(n-1)v_y\right)^2 + \sigma^2 \left(v^2_y + v^2_x\right)}. \tag{42}
\]

Our calibration generates an annual expected excess return of $r_{y,t} = 3.44\%$ and volatility $\sigma_{y,t} = 9.71\%$ for the claim to EBIT. As we will see below, this will generate realistic excess returns and volatility for the claim to dividends, which is a leveraged claim.

Figure 5 shows that the term structure of EBIT volatility, and the term structures of expected returns and volatilities of EBIT strips, are upward sloping. This result is not surprising given that the form of the EBIT dynamics is the same as that of the dividend dynamics in the baseline BY model of the previous section.
B.1. Leverage Dynamics

We now close the model by specifying leverage dynamics and an accounting identity to endogenously determine the dividend dynamics of the aggregate representative firm. Recall from equation (33) that enterprise value takes the form

\[ V(y_t, x_t, \sigma_t) \approx e^{y_t + U_0 + U_x x_t + U_\sigma \sigma_t^2}. \]  

(43)

We specify that at all dates \( t \) the firm issues riskless debt that matures at date \((t + 1)\) with present value equal to

\[ B(\ell_t, y_t, x_t, \sigma_t) = e^{\ell_t + y_t + U_0 + U_x x_t + U_\sigma \sigma_t^2} \approx e^{\ell_t} V(y_t, x_t, \sigma_t). \]  

(44)

We interpret \( e^{\ell_t} \approx B(\ell_t, y_t, x_t, \sigma_t) \) as the leverage of the firm. Since the debt is presumed to be riskless, the firm must pay \( e^{r(f)(x_t, \sigma_t)} B(\ell_t, y_t, x_t, \sigma_t) \) at date \((t + 1)\). It does so by issuing at this time debt with face value \( B(\ell_{t+1}, y_{t+1}, x_{t+1}, \sigma_{t+1}) \), with all residual cash flows paid out as dividends. As such, dividends \( D(t+1) \) paid out at date \((t + 1)\) are

\[ D(t + 1) = B(\ell_{t+1}, y_{t+1}, x_{t+1}, \sigma_{t+1}) - e^{r(f)(x_t, \sigma_t)} B(\ell_t, y_t, x_t, \sigma_t) + e^{y_{t+1}}. \]  

(45)

Equation (45) is an accounting identity that equates cash inflows to outflows, and is analogous to equation (1) in our binomial setting.17

We choose the dynamics of log-leverage so that (i) it is mean-reverting, and (ii) in the continuous time limit of the model, bond issuances are locally deterministic. We emphasize that these conditions are chosen for parsimony as they reduce to only four \((\ell, \rho_\ell, \rho_{\ell x}, \rho_{\ell \sigma})\) the number of free parameters in leverage dynamics (and no free volatility parameters), thus imposing a lot of discipline on the model.18 In particular, we choose

\[ \ell_{t+1} = - \bar{\ell} + \rho_\ell (\ell_t - \bar{\ell}) + \rho_{\ell x} x_t + \rho_{\ell \sigma} (\sigma_t^2 - \bar{\sigma}^2) + v_x \sigma_t \varepsilon_{x,t+1} - v_\sigma \sigma_t \varepsilon_{\sigma,t+1}. \]  

(46)

We set the parameters \( \{\bar{\ell} = \log(0.35), \rho_\ell = 0.985\} \) to closely match the empirical estimates for log-leverage dynamics discussed in the empirical section.19 We

---

17 Recall the risk-free rate is zero in the binomial setting. This explains the difference between the two equations.

18 We also emphasize that by imposing dividend dynamics to be locally deterministic in the continuous-time limit, that is, of the form \( D dt \), not only are we forcing our model to look like those used in the rest of the literature, but we are also making it more difficult to generate a downward-sloping term structure of dividend VRs. Indeed, dividend dynamics that are not locally deterministic have very high short-horizon volatilities.

19 Note that in this specification the leverage ratio can in principle be greater than one, in conflict with the assumption that debt is risk-free. We chose this specification for parsimony to obtain closed-form solutions. We find that leverage ratios above one happen very rarely in our benchmark calibration. Moreover, we can show that this feature is not driving our results in that...
choose \( \{ \rho_\nu = 0.2, \rho_\sigma = -10.0 \} \) for our calibration. This specification implies that dividends follow the endogenously determined process

\[
D(t + 1) = e^{y_t} \left[ e^{\tilde{\tau} + \rho_\nu (\ell_t - \bar{\tau}) + \rho_\sigma x_t + \rho_{\sigma^2} (\sigma^2 - \tilde{\sigma}^2 ) + \mu_y + \rho_y x_t + U_0 + U_s \rho_s x_t + U_s (\sigma^2 (1 - \rho_s) + \rho_s \sigma^2 ) ) (47) \right.
\]

\[
- e^{\ell_t + U_0 + U_s \sigma^2 + \tilde{\gamma}_s + \frac{1}{2} \tilde{\sigma}_s + \tilde{\gamma}_s} + e^{(\mu_y + \rho_y x_t + \nu_y \sigma_t z_{t+1} + \nu_y \sigma_t z_{t+1})} \right]
\]

\[
= D_t (t + 1) - D_{t+1} (t + 1) + e^{\gamma_{t+1}}. \tag{48}
\]

Since all terms inside the bracket follow stationary processes, it follows that the dividend-EBIT ratio is also stationary. It is therefore not surprising that the volatility of dividends at long horizons approaches the volatility of EBIT, as we show in the top left panel in Figure 6. Importantly, note that the term structure of dividend volatility is downward sloping, consistent with the empirical evidence. \(^{20}\)

### B.2. Dividend Strips

Here we provide a closed-form expression for the price of dividend strips, defined as

\[
V_d^T (t, \ell_t, y_t, x_t, \sigma_t) = E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i})} D(T) \right]
\]

\[
= E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i})} (D_t (T) - D_{t+1} (T) + e^{\gamma_T}) \right]. \tag{49}
\]

The price of dividend strips is a sum of three terms, each of which can be expressed in an exponential-affine form. The first term is

\[
V_d^T (t, \ell_t, y_t, x_t, \sigma_t) = E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i})} B(\ell_T, y_T, x_T, \sigma_T) \right]. \tag{50}
\]

The second term is

\[
V_{d2}^T (t, \ell_t, y_t, x_t, \sigma_t) = E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i})} B(\ell_{T-1}, y_{T-1}, x_{T-1}, \sigma_{T-1}) e^{m_T} (x_{T-1}, \sigma_{T-1}) \right]
\]

\[
= E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i})} B(\ell_{T-1}, y_{T-1}, x_{T-1}, \sigma_{T-1}) E_{T-1} \left[ e^{m_T + \gamma_T} (x_{T-1}, \sigma_{T-1}) \right] \right]
\]

\[
= E_t \left[ e^{(\sum_{i=1}^{n} m_{t+i})} B(\ell_{T-1}, y_{T-1}, x_{T-1}, \sigma_{T-1}) \right]
\]

\[
= V_{d1}^{T-1} (t, \ell_t, y_t, x_t, \sigma_t). \tag{51}
\]

imposing reflecting boundaries on the leverage process (i.e., restricting the leverage ratio to be between 20% and 65%) and solving the model via Monte Carlo produces results very similar to those reported here.

\(^{20}\) In this section we report the term structure of dividends estimated at the long-run mean of the state vector. This allows us to make use of the closed-form solutions for all the prices and moments of returns. In contrast, in Section I we investigate dividends that are time-aggregated over a period of one year. In unreported results we show that aggregating dividends in this model does not significantly impact our results.
The third term is the claim to the EBIT strip, $V_T^d(t, y_t, x_t, \sigma_t)$. From the law of iterated expectations, the first term satisfies

$$V_{d,1}^T(t-1, \ell_{t-1}, y_{t-1}, x_{t-1}, \sigma_{t-1}) = E_{t-1} \left[ e^{\bar{y}_t} V_{d,1}^T(t, \ell_t, y_t, x_t, \sigma_t) \right].$$

(52)

Because state vector dynamics are affine, we know the solution is of the form

$$V_{d,1}^T(t, \ell_t, y_t, x_t, \sigma_t) = e^{\bar{y}_t + F_0(n) + F_\ell(n) \ell_t + F_x(n) x_t + F_\sigma(n) \sigma_t^2}$$

(53)

$$V_{d,1}^T(t-1, \ell_{t-1}, y_{t-1}, x_{t-1}, \sigma_{t-1}) = e^{\bar{y}_{t-1} + F_0(n+1) + F_\ell(n+1) \ell_{t-1} + F_x(n+1) x_{t-1} + F_\sigma(n+1) \sigma_{t-1}^2}$$

(54)

The final conditions are \{ $F_0(0) = U_0$, $F_\ell(0) = 1$, $F_x(0) = U_x$, $F_\sigma(0) = U_\sigma$ \}.
Plugging equations (53) and (54) into equation (52), performing the expectation, and then collecting terms independent of the state vector and linear in \((\ell_t, x_t, \sigma^2_t)\), we obtain the four recursive equations for \(\left(F_\ell(n), F_x(n), F_0(n), F_\sigma(n)\right)\).

To characterize the risk premia of dividend strips, define the gross excess return on dividend strips as
\[
\tilde{R}_{d,t+1} = \frac{V_{d,t+1}}{R_f(t)}
\]
(55)

The term structures of excess returns and return variances have closed-form expressions and are displayed in the top right and middle left panels of Figure 6. In contrast to the original BY model, these term structures are downward sloping, in line with the empirical findings of BBK. Intuitively, this result occurs because dividend policies that are consistent with stationary leverage ratios tend to shift risk from the long horizon toward the short horizon (as exhibited by the downward-sloping term structure of dividend volatility).

**B.3. Equity Returns**

The value of equity is equal to the sum of all dividend strips:
\[
V_d(\ell_t, y_t, x_t, \sigma_t) = \sum_{T=t+1}^\infty V_{d,T}(t, \ell_t, y_t, x_t, \sigma_t)
\]
\[
= \sum_{T=t+1}^\infty \left[ V_{d,1,T}(t, \ell_t, y_t, x_t, \sigma_t) - V_{d,2,T}(t, \ell_t, y_t, x_t, \sigma_t) + V_{y,T}(t, \ell_t, y_t, x_t, \sigma_t) \right]
\]
(56)

where we have used equations (50) and (51) to find
\[
\sum_{T=t+1}^\infty \left[ V_{d,1,T}(t, \ell_t, y_t, x_t, \sigma_t) - V_{d,2,T}(t, \ell_t, y_t, x_t, \sigma_t) \right] = -B(\ell_t, y_t, x_t, \sigma_t).
\]
(57)

This equation is intuitive—it states that the value of equity equals enterprise value minus the value of debt outstanding, and follows from Modigliani–Miller’s capital structure irrelevance theorem.

The excess return on equity is approximately equal to its EBIT counterpart scaled by a leverage factor (with a convexity correction term):
\[
\bar{r}_{d,t} - r_{f,t} \approx \left( \frac{1}{1 - e^{\ell_t}} \right) \left( \bar{r}_{y,t} - r_{f,t} - \frac{\sigma^2_{y,t}}{2} \left( \frac{e^{\ell_t}}{1 - e^{\ell_t}} \right) \right).
\]
(58)
Hence, the expected excess return and return volatility are

\[
\bar{r}_{d,t} \approx \left( \frac{1}{1-e^{\ell_t}} \right) \left[ v_y^2 \kappa_y U_y \lambda_y + (v_y \lambda_c + v_x \kappa_y U_x \lambda_x) \sigma_t^2 - \frac{\sigma_t^2}{2} \left( \frac{e^{\ell_t}}{1-e^{\ell_t}} \right) \right]
\]

\[+ \frac{1}{2} \sigma_{d,t}^2 \]

\[
\sigma_{d,t} = \left( \frac{1}{1-e^{\ell_t}} \right) \sqrt{(\kappa_y U_y \nu_\gamma)^2 + \sigma_t^2 \left[ v_y^2 + v_x^2 + (\kappa_y U_x \nu_\gamma)^2 \right]}
\]

This produces reasonable estimates for the equity premium, equity volatility, and Sharpe ratio (\(Sh\)):

\[
\bar{r}_d(\ell_t = \bar{\ell}, \sigma_t = \bar{\sigma}) = 6.01\%.
\]

\[
\sigma_d(\ell_t = \bar{\ell}, \sigma_t = \bar{\sigma}) = 14.94\%.
\]

\[
Sh_d(\ell_t = \bar{\ell}, \sigma_t = \bar{\sigma}) = 0.40.
\]

As shown in Figure 6, the excess returns and return volatility for the stock index are well below the excess returns and volatilities of short-maturity dividend strips, consistent with the findings of BBK.

**B.4. Comparative Statics: Implications for the Cross-Section**

Although the focus of the paper is on the analysis of aggregate asset prices, the economic mechanism linking dividends to capital structure has implications that can be investigated using cross-sectional data. Here, we discuss the most relevant implications of the model by performing comparative statics.

All parameters in the leverage process in equation (46) have a significant impact on the quantitative results. However, the crucial parameter is the persistence parameter \(\rho\). This is demonstrated in Figure 7. This figure shows that even a relatively small change in the persistence parameter from the benchmark value \(\rho = 0.985\) to \(\rho = 0.98\) (i.e., a less persistent leverage ratio process) significantly impacts both the level of short-horizon volatility and the slope of the term structure of volatility.

The cross-sectional implications of the previous analysis are that those firms that maintain a tighter leverage target will have a more downward-sloping quantity of risk and in turn short-horizon dividend strips with higher excess returns and return volatilities. We would like to qualify this prediction, however, by noting that, in maintaining the spirit of endowment economies, we have ignored investment. Moreover, we have ignored the possibility that firms issue and repurchase equity. (As we show in Section II this issue is not minor, as the short-horizon dividend volatility is approximately one-half the volatility of all payouts to shareholders.) These additional features would no doubt impact individual firms’ dividend dynamics. As such, we feel it would be more prudent when attempting to make cross-sectional predictions about dividend strip returns to focus directly on the term structures of dividend volatilities.
(which can readily be estimated) across firms rather than just their leverage policies.

C. Calibrating the Dividend Process to Match Additional Empirical Moments

In the previous section, we specify the leverage process in equation (46) to be stationary, and then use the equation

\[ D_{t+1} = B_{t+1} + Y_{t+1} - e^{\rho L} B_t \]  

(62)

to identify the dividend process. The advantage of this approach is that the dynamics are intuitive and provide analytic solutions. However, the model fails empirically on a few dimensions. For example, the model predicts a strong correlation between changes in log-dividends and market returns over short horizons (e.g., one year), in contrast to empirical estimates of a very low correlation as reported in Section I above. As shown in the bottom left and right panels in Figure 6, this high correlation generates large CAPM betas and negative CAPM alphas for short-horizon strips, in contrast to the empirical findings of BBK and BHKV. It also generates flat Sharpe ratios.

As such, in this section we investigate the properties of an alternative and more flexible dividend process that can be calibrated to match stationary leverage ratios, downward-sloping term structures for dividend strips, and a low correlation between short-horizon dividends and stock returns (which in turn implies low CAPM betas for short-horizon strips). Moreover, we calibrate the model so that short-horizon dividend strips have high CAPM alphas and high Sharpe ratios, even though we remain agnostic regarding the strength of these
In particular, we directly specify the dividend process in terms of the current log-leverage level \( \ell_t \), and then use equation (62) to identify the leverage process. To generate stationary leverage ratios, we specify a process where the target dividend-EBIT ratio is a decreasing function of leverage—that is, when leverage is high (low), the dividend-EBIT ratio target is low (high), which pushes leverage ratios back toward their target value. In particular, for \( c_1 > 0 \), we specify the dividend process in the spirit of Lintner (1956) and Lambrecht and Myers (2012) as follows:

\[
\Delta d_t = \rho_d \left[ y_t + c_0 + c_1 (\ell - \ell_t) - d_t \right] + a_c \sigma_c \bar{\epsilon}_c + a_y \sigma_y \bar{\epsilon}_y + a_x \sigma_x \bar{\epsilon}_x + a_\nu \sigma_\nu \bar{\epsilon}_\nu \tag{63}
\]

We choose the calibration \( \rho_d = 0.03 \), \( c_0 = -0.1 \), \( c_1 = 1 \), \( a_c = 6.5 \), \( a_y = -4.8 \), \( a_\nu = 6.6 \), and \( a_\sigma = -12,000 \).\(^{22}\) In addition, we assume that, if current leverage reaches a high (low) value, then the firm sells (buys) assets at market value to maintain leverage values below (above) this maximum (minimum).\(^{23}\) Because the transactions are fairly priced, they have no impact on the value of equity. We describe this mechanism further in the Internet Appendix.

As shown in the top left and right panels in Figure 8, this framework generates mostly downward-sloping term structures of excess returns and volatilities for dividend strips. It also matches the empirical estimates of the average log aggregate leverage ratio (−0.95 versus −0.96) and the one-year autocorrelation (0.91 versus 0.87 empirically). In addition, although this model loses analytic tractability because the implicit dynamics for \( \ell_t \) are no longer affine, it provides the flexibility needed to match other empirical moments reported by BBK. For example, Figure 8 shows that the short-horizon dividend strips have CAPM betas that are significantly less than one (and large CAPM alphas) in spite of having large Sharpe ratios, consistent with BBK.

To understand the previous results, note that, since the one-period dividend strip is not impacted by the dynamics of leverage, we can obtain its return and CAPM beta in closed form. Indeed, we can express its log-return as

\[
r_t^{d,t+1} - r_f,t = a_c \sigma_c \bar{\epsilon}_c + a_y \sigma_y \bar{\epsilon}_y + a_\nu \sigma_\nu \bar{\epsilon}_\nu - \frac{\sigma^2}{2} \left( a_c^2 - 2a_c \lambda_c \right) - \frac{\sigma^2}{2} \left( a_y^2 - 2a_y \lambda_y \right) - \frac{\sigma^2}{2} \left( a_\nu^2 - 2a_\nu \lambda_\nu \right) \tag{64}
\]

\(^{21}\) As reported in BBK, the \( t \)-statistics on these moments are small. In addition, BBK do not account for any relative tax disadvantage of dividends versus capital gains, which would tend to reduce the excess returns of the short-horizon strips (see Schulz (2013) for a detailed analysis of the effect of taxes on the properties of dividend strips). See also Boguth et al. (2011) for other concerns.

\(^{22}\) Note that the two constant terms \( (c_0 + c_1 \ell) \) show up only as a sum in equation (63), and hence are not separately identifiable until information on the endogenously obtained leverage process is collected. As such, \( c_b \) is calibrated so that \( \ell \) matches the actual steady-state log-leverage estimated in population. Separately, note that, while \( a_\nu = -12,000 \) appears to be a large number, the appropriate comparison of the other factor loadings \( a_i \) with the factor loading on \( \bar{\epsilon}_\nu \) is the combination of terms \( (\frac{\sigma_c^2}{2}) \approx 4.7 \). Hence, this factor loading is of the same size as the others.

\(^{23}\) Such transactions occur infrequently, and are not generating our results. We impose a minimum leverage ratio of 25% and a maximum leverage ratio of 65%.
Hence, the log expected excess return and the return volatility for the first strip are

\[
\begin{align*}
\bar{r}(V_d^{t+1}(t)) &= E \left[ r_{d,t}^{t+1} - r_{f,t} \right] + \frac{1}{2} \text{Var} \left[ r_{d,t}^{t+1} - r_{f,t} \right] \\
&= \sigma_t^2 \left( \lambda_c \sigma_c + \lambda_y \sigma_y \right) + \nu^2 \sigma_c \lambda_c \\
\sigma(V_d^{t+1}(t)) &= \sqrt{\sigma_t^2 \left( \alpha_c^2 + \alpha_y^2 + \alpha_x^2 \right) + \alpha_x^2 \nu^2}.
\end{align*}
\]

At the steady-state values, we find the following annualized values:

\[
\begin{align*}
\bar{r} \left( V_d^{t+1}(t, \ell, x_t = \bar{x}, \sigma_t = \bar{\sigma}) \right) &= 14.0\% \quad (66) \\
\sigma \left( V_d^{t+1}(t, \ell, x_t = \bar{x}, \sigma_t = \bar{\sigma}) \right) &= 27.7\% \quad (67)
\end{align*}
\]
Dividend Dynamics and the Term Structure of Dividend Strips

\[ \text{Sh} \left(V_d^{t+1}(t, \ell, x, \sigma) = 0, \sigma_i = \bar{\sigma}) \right) = 0.51. \] (68)

Note that the Sharpe ratio of the short-horizon dividend strip is significantly higher than the Sharpe ratio of the index, which is 0.40, as reported in equation (61).

In addition to having analytic solutions for its return moments, we can also identify the CAPM beta for the short-horizon strip. Indeed, recall from equation (58) that the stochastic component of the stock return is equal to the stochastic component of the enterprise value return, scaled up by a leverage factor:

\[ r_d, t^\text{stoch} = \left(1 - e^{\ell t}\right) [k_1 y (U_x v_y \sigma_y + U_v v_y \bar{\epsilon} + v_x \sigma_x + v_y \bar{\epsilon} + v_y \sigma_y), \] (69)

where we use equation (32) in the last line. We can therefore determine the covariance between returns for the first dividend strip and the market portfolio:

\[ \text{Cov} \left[r_d^{t+1}, r_m \right] = \left(1 - e^{\ell t}\right)^2 [(k_1 v_y \sigma_y)^2 + (k_1 v_y \sigma_y)^2 + (v_y \sigma_y)^2 + (v_y \sigma_y)^2]. \] (70)

\[ \text{Var} \left[r_d \right] = \left(1 - e^{\ell t}\right)^2 [(k_1 v_y \sigma_y)^2 + (k_1 v_y \sigma_y)^2 + (v_y \sigma_y)^2 + (v_y \sigma_y)^2]. \] (71)

Dividing equation (70) by equation (71) provides an estimate of the CAPM beta, which for our parameterization equals 0.49. Intuitively, if we want to calibrate the model so that short-horizon dividend strips have low CAPM betas and high Sharpe ratios, we should choose the factor loading vector \((a_c, a_y, a_x, a_\sigma)\) to be more collinear with the pricing kernel than the factor loading vector on stock returns. Further, if we want short-horizon dividend strips to have low CAPM betas, then we should choose a low covariance between their factor loading vectors. As reported in Figure 8, the previous calibration allows us to generate short-horizon dividend strips with CAPM betas that are significantly less than one (and large CAPM alphas) in spite of having large Sharpe ratios, consistent with BBK.

**D. Discussion**

We have shown above that, if we retain the pricing kernel of BY, but replace their proposed dividend process with a process that combines an unleveraged cash flow (i.e., EBIT) process (with the same functional form as the dividend processes in BY) with a dynamic capital structure strategy that produces stationary leverage ratios, then the model generates a downward-sloping term structure of dividend volatilities, consistent with empirical findings. In addition, consistent with results reported in BBK, short-maturity dividend strip returns have higher expected excess returns and higher volatilities than stock returns. Moreover, the model can be calibrated to match the findings of BHKV.
that short-maturity dividend strips have low CAPM betas yet high CAPM alphas. The downward-sloping term structure of dividend strip returns is due to the implicit divestments (investments) that the firm imposes in good (bad) times on stockholders via capital structure decisions, which generates stationary leverage ratios. As such, long-maturity dividend strips are not as risky as typically imagined—rather, they are about as risky as long-maturity EBIT strips, since dividends and EBIT are cointegrated. However, claims to all future dividends (i.e., equity) are riskier than claims to EBIT (i.e., equity plus debt). The implication is that dynamic capital structure decisions that generate stationary leverage ratios shift the risk in dividends from long horizons to short horizons.

Interestingly, this model also generates long-horizon “excess volatility” in stock return volatility $\sigma^Y_{sv} \approx 0.15$ is higher than long-horizon dividend volatility $\sigma^D_{sv} \approx 0.12$. This prediction is more in line with observation compared to the BY model, which predicts that long-horizon dividend volatility is larger than stock volatility. The intuition for this result is that, since dividends are cointegrated with EBIT in the model, its long-horizon volatility is equal to the long-horizon volatility of (unleveraged) EBIT. In contrast, stock return volatility is pushed up by a “leverage factor” $(\frac{1}{1-L})$, because, in the short and medium horizons, the dividend process is a levered claim.

### IV. Habit Formation Model with Stationary Leverage Ratios

We now incorporate stationary leverage ratios in a version of the CC habit formation model. In contrast to the BY model, which specifies cash flow dynamics with a predictable component, the framework we consider here has no predictability in aggregate cash flows, but generates predictability in excess returns via time-variation in risk premia. Specifically, we assume that cash flows follow an i.i.d. process, and that shocks to the market price of risk are negatively correlated with shocks to these cash flows. But rather than modeling dividend dynamics exogenously as in CC, we instead combine i.i.d. EBIT dynamics with a dynamic capital structure policy that generates stationary leverage ratios in order to endogenously determine dividend dynamics. Because much of the theory is very similar to the BY framework of the previous section, we present here only the main results, and relegate the derivations to the Internet Appendix. To isolate the impact of stationary leverage ratios on the overall results, we also investigate a particular parameterization of the CC model in which the price of risk is set to be constant (which corresponds to a simple dynamic Gordon growth model with time-varying leverage).

#### A. Habit Model

The CC habit formation model assumes that cash flow dynamics follow an arithmetic Brownian motion, and reverse engineers the structure of the habit so that the risk-free rate is constant. Here, for tractability, we start with a
pricing kernel that captures many of these features. In particular, we specify the dynamics for the log pricing kernel, the market price of risk, and log-EBIT as follows:

\[ m_{t+1} = -r_f - \frac{1}{2} \theta_t^2 - \theta_t \tilde{e}_{m,t+1} \] (72)

\[ \theta_{t+1} = \bar{\theta} (1 - \rho_\theta) + \rho_\theta \theta_t - \sigma_\theta \tilde{e}_{m,t+1} \] (73)

\[ \Delta y_{t+1} = \mu_y + v_{ym} \tilde{e}_{m,t+1} + v_y \tilde{e}_{y,t+1}. \] (74)

The time interval is monthly. We investigate the following calibration:

- \( r_f = 0.002 \), \( \bar{\theta} = \frac{0.5}{\sqrt{12}} \), \( \rho_\theta = 0.98 \), \( \sigma_\theta = 0.01 \), \( \mu_y = 0.0015 \), \( v_{ym} = 0.011 \), and \( v_y = 0.02 \).

Note that, as in CC, innovations in the pricing kernel and the market price of risk are (conditionally) perfectly correlated.

We approximate the log of enterprise value to EBIT via

\[ z_{y,t} \equiv \log (V_{y,t}/Y_t) \approx U_0 - U_\theta \theta_t. \] (75)

We also approximate the log return to the EBIT claim via

\[ r_{y,t+1} \approx \kappa_{by} + \kappa_{1y} z_{y,t+1} - z_{y,t} + \Delta y_{t+1}, \] (76)

where \( y_t = \log Y_t \) and where \( \kappa_{by}, \kappa_{1y} \) are given in the Internet Appendix in terms of \( \bar{z}_y = U_0 - U_\theta \bar{\theta} \). The five parameters \{\( \bar{z}_y, \kappa_{by}, \kappa_{1y}, U_0, U_\theta \}\) are determined from these equations and the following first-order condition:

\[ 0 = E_t [m_{t+1} + r_{y,t+1}] + \frac{1}{2} \text{Var}_t [m_{t+1} + r_{y,t+1}]. \] (77)

A.1. Enterprise Value Return

It follows from equation (76) that the log expected excess return on the EBIT claim is

\[ r_{y,t} \equiv \log E_t \left[ e^{r_{y,t+1}-r_f} \right] = v_{ym} \theta_t \left( \frac{1^{1-k_{by} \rho_\theta}}{1-k_{by} \rho_\theta - k_{1y} \sigma_\theta} \right), \] (78)

and the volatility on the EBIT claim is

\[ \sigma_{y,t} = \sqrt{\log E_t \left[ e^{2(r_{y,t+1}-r_f)} \right] - 2 \log E_t \left[ e^{r_{y,t+1}-r_f} \right]} \]

\[ = \sqrt{(\kappa_{by} U_\theta \sigma_\theta + v_{ym})^2 + v_y^2}. \] (79)
Our calibration generates an annual expected excess return of $\bar{r}_{yt} = 3.4\%$ and volatility $\sigma_{yt} = 9.7\%$ for the claim to EBIT. As we will see below, this implies a realistic excess return and volatility for the claim to dividends, which is a leveraged claim.

A.2. Term Structure of EBIT

Because log-EBIT follows an arithmetic Brownian motion process, the term structure of dividend expected growth rates is flat over all horizons $n$:

$$g_{yt,n} = \frac{1}{n} \log (E_t [e^{\bar{y}_T - \bar{y}_t}]) = \mu_y + \frac{1}{2}\left(v_{ym}^2 + v_y^2\right) \quad \forall n.$$ (80)

For our calibration, we find the annualized expected growth to be $g_{yt,n} = 2.1\%$ for all $n$.

Similarly, the term structure of dividend volatilities is also flat over all horizons:

$$\sigma_{yt,n} = \sqrt{\left(\frac{1}{n}\right) \left(\log E_t [e^{2(\bar{y}_T - \bar{y}_t)}] - 2 \log E_t [e^{\bar{y}_T - \bar{y}_t}]\right)} = \sqrt{v_{ym}^2 + v_y^2} \quad \forall n.$$ (81)

For our calibration, we find the annualized volatility to be $\sigma_{yt,n} = 7.9\%$ for all $n$.

A.3. EBIT Strips

The date $t$ claim to the EBIT strip $e^{\bar{y}_T}$ paid out at date $T$ is defined as

$$V_y^T(t, y_t, \theta_t) = E_t \left[e^{\sum_{i=1}^{n} m_{yi} + y_T}\right].$$ (83)

Note that from the law of iterated expectations we have

$$V_y^T(t - 1, y_{t-1}, \theta_{t-1}) = E_{t-1} \left[e^{m_{n} V_y^T(t, y_t, \theta_t)}\right].$$ (84)

Since the state vector dynamics are affine, the solution is of the form

$$V_y^T(t, y_t, \theta_t) = e^{y_T + U_y(n) - U_y(n\theta_t)}. \quad (85)$$

In the Internet Appendix we derive the recursive equations for $(U_y(n), U_y(n\theta_t))$.

Define the date $t$ one-period gross return on a strip that matures at date $T$ via

$$\tilde{R}^T_{yt} = e^{\bar{r}_{yt+1}} = \frac{V_y^T(t+1, y_{t+1}, \theta_{t+1})}{V_y^T(t, y_t, \theta_t)}.$$ (86)
In the Internet Appendix we show that the log expected excess returns for the EBIT strips are 

\[ \bar{r}(V^T(t)) = \log E_t \left[ e^{(\bar{r}^T_{y,t+1} - r_f)} \right] = \theta_t \left( \sigma_y U_y (n - 1) + \nu_{ym} \right). \]  

(87)

We also define EBIT strip volatility as 

\[ \sigma(V^T(t)) = \sqrt{\log E_t \left[ e^{2(\bar{r}^T_{y,t+1} - r_f)} \right] - 2 \log E_t \left[ e^{(\bar{r}^T_{y,t+1} - r_f)} \right]} = \sqrt{(\sigma_y U_y (n - 1) + \nu_{ym})^2 + \nu_y^2}. \]  

(88)

We plot the term structures of excess returns and return volatilities for the EBIT strips in top left and right panels in Figure 9. As noted in BBK, both are upward sloping.

### A.4. Leverage Dynamics

Recall from equation (75) that 

\[ V_y(y_t, \theta_t) \approx e^{y_t + U_0 - U_y \theta_t}. \]  

(89)

As in the modified BY model, we assume that at all dates \( t \) the firm issues riskless debt that matures at date \( (t + dt) \) with present value equal to 

\[ B(\ell_t, y_t, \theta_t) = e^{\ell_t + y_t + U_0 - U_y \theta_t} \approx e^{\ell_t} V_y(y_t, \theta_t). \]  

(90)

As before, we interpret \( e^{\ell_t} \approx \frac{B(\ell_t, y_t, \theta_t)}{V_y(y_t, \theta_t)} \) as the leverage of the firm. Since it is riskless, the firm must pay \( e^{\bar{r}} B(\ell_t, y_t, \theta_t) \) at date \( (t + 1) \). It does so by issuing at this time debt with face value \( B(\ell_{t+1}, y_{t+1}, \theta_{t+1}) \), with all residual cash flows paid out as dividends. As such, dividends \( D(t + 1) \) paid out at date \( (t + 1) \) are 

\[ D(t + 1) = B(\ell_{t+1}, y_{t+1}, \theta_{t+1}) - e^{\bar{r}} B(\ell_t, y_t, \theta_t) + e^{y_{t+1}} \equiv D_1(t + 1) - D_2(t + 1) + e^{y_{t+1}}. \]  

(91)

We specify log-leverage dynamics so that leverage is mean-reverting and bond issuances are locally deterministic in the continuous-time limit: 

\[ \ell_{t+1} = \ell_t + \rho_\ell (\ell_t - \bar{\ell}) + \rho_{\ell \theta} (\theta_t - \bar{\theta}) - \nu_{ym} \bar{e}_{m_{t+1}} - \nu_y \bar{e}_{y_{t+1}} - U_y \sigma_y \bar{e}_{m_{t+1}}. \]  

(92)

This specification leaves us with no free volatility parameters and three free drift parameters. We choose the calibration \( \bar{\ell} = \ln(0.35) \), \( \rho_\ell = 0.985 \), and \( \rho_{\ell \theta} = -0.02 \).
Figure 9. Term structures of EBIT and dividends in the modified CC framework. The top left panel shows the EBIT strips’ excess returns, the top right panel shows the EBIT strips’ return volatilities (standard deviation), the bottom left panel shows the dividend strips’ excess returns, and the bottom right panel shows the dividend strips’ return volatility in the modified CC framework. For comparison, note that the annual expected excess return and return volatility for the claim to EBIT is $r_{y,t} = 3.4\%$ and $\sigma_{y,t} = 9.7\%$, respectively. In addition, the annual expected excess return and return volatility for the claim to dividends (equity) is $r_{d,t} = 5.23\%$ and $\sigma_{d,t} = 14.92\%$, respectively. The term structures are evaluated at the long-run value ($\theta_t = \theta$).

A.5. Dividend Strips

Here we provide a closed-form expression for the price of dividend strips, defined as

$$V_d^T(t, \ell, y_{\cdot}, \theta_{\cdot}) = \mathbb{E}_t \left[ e^{\left(\sum_{i=1}^{m} m_{i,t} \right)} D(T) \right] = \mathbb{E}_t \left[ e^{\left(\sum_{i=1}^{m} m_{i,t} \right)} (D_1(T) - D_2(T) + e^{y_T}) \right].$$

(93)

The price of dividend strips is a sum of three terms, each of which can be expressed in an exponential-affine form. The first term is

$$V_{d1}^T(t, \ell, y_{\cdot}, \theta_{\cdot}) = \mathbb{E}_t \left[ e^{\left(\sum_{i=1}^{m} m_{i,t} \right)} B(\ell_{\cdot}, y_{\cdot}, \theta_{\cdot}) \right].$$

(94)
The second term is
\[ V_{d_2}^T(t, \ell_t, y_t, \theta_t) = E_t \left[ e^{(\sum_{i=1}^{n} m_{i,t})} B(\ell_{T-1}, y_{T-1}, \theta_{T-1}) e^{r_f} \right] 
= E_t \left[ e^{(\sum_{i=1}^{n-1} m_{i,t})} B(\ell_{T-1}, y_{T-1}, \theta_{T-1}) e^{m_{T-1}+r_f} \right] 
= E_t \left[ e^{(\sum_{i=1}^{n-1} m_{i,t})} B(\ell_{T-1}, y_{T-1}, \theta_{T-1}) \right] \]
\[ = V_{d_2}^{T-1}(t, \ell_{T-1}, y_{T-1}, \theta_{T-1}). \] (95)

The third term is the claim to the EBIT strip, \( V_y^T(t, y_t, \theta_t). \)

From the law of iterated expectations, the first term satisfies
\[ V_{d_1}^T(t - 1, \ell_{t-1}, y_{t-1}, \theta_{t-1}) = E_{t-1} \left[ e^{m_{d_1}} V_{d_1}^T(t, \ell_t, y_t, \theta_t) \right]. \] (96)

Because state vector dynamics are affine, we know the solution is of the form
\[ V_{d_1}^T(t, \ell_t, y_t, \theta_t) = e^{y_t+F_0(n)+F_1(n)\ell_t-F_0(n)\theta_t} \quad \text{(97)} \]
\[ V_{d_1}^T(t - 1, \ell_{t-1}, y_{t-1}, \theta_{t-1}) = e^{y_{t-1}+F_0(n+1)+F_1(n+1)\ell_{t-1}-F_0(n+1)\theta_{t-1}}. \] (98)

The final conditions are \( F_0(0) = U_0, F_1(0) = 1, \) and \( F_0(0) = U_0. \)

Plugging equations (97) and (98) into equation (96), performing the expectation, and then collecting terms independent of the state vector and linear in \((\ell_t, \theta_t),\) we obtain recursive equations for the three functions \( F_0(n), F_1(n), F_0(n+1) \). We report them in the Internet Appendix.

We define the gross excess return on dividend strips as
\[ \frac{R_{d_1}^{T+1}}{R_f} = \frac{V_{d_1}^T(t+1)}{V_{d_1}^T(t)} \]
\[ = \left( \frac{1}{V_{d_1}^T(t)} \right) \left[ V_{d_1}^T(t+1) - V_{d_2}^T(t + 1) + V_y^T(t + 1) \right]. \] (99)

The first and second moments have analytic expressions, and are given in the Internet Appendix. We plot their term structures in the bottom left and right panels in Figure 9. As in the case of BY, we see that, even though the term structures of excess returns and volatilities for EBIT strips are upward sloping, once we combine EBIT dynamics with a leverage process that generates stationary leverage ratios, the endogenously determined dividend dynamics generate term structures of excess returns and volatilities for dividend strips that are downward sloping, in agreement with BBK.

**B. A Special Case: Constant Market Price of Risk**

In the modified CC model, the aggregate risk premium varies due to both time-varying price of risk as well as time-varying leverage. To isolate the quantitative impact of time-varying leverage on the properties of dividends and
dividend strips, we consider a special calibration of the CC model in which we shut down time-varying price of risk. Hence, in this special case, all of the time-variation in the model comes from changes in the leverage process. We use this alternative calibration to show that it is the time-varying leverage process that allows us to match the properties of dividends and dividend strips in the data.

Setting \( \sigma_n = 0 \) in the CC framework of Section IV. A generates a constant market price of risk. The claim to EBIT then satisfies the Gordon growth formula,

\[
V_y(t) = Ae^{\gamma},
\]

where the constant is

\[
A = \frac{e^{-r_f + \mu_y + \frac{\nu_m^2}{2} + \frac{\nu^2}{2} - \nu y \theta}}{1 - e^{-r_f + \mu_y + \frac{\nu_m^2}{2} + \frac{\nu^2}{2} - \nu y \theta}}.
\]

To more closely match historical equity premium and stock volatility, we set \( \nu_{ym} = 0.023 \) and \( \nu_y = 0.01 \). This calibration generates an annualized excess return for the enterprise value of \( \bar{r} = 4.0\% \) and volatility \( \sigma_r = 8.7\% \). The i.i.d. dynamics for EBIT growth imply that the term structures of expected growth and volatilities for EBIT are flat. Moreover, the constant market price of risk generates flat term structures for excess returns and volatilities of EBIT strips that are equivalent to the values on the EBIT claim:

\[
\begin{align*}
g_{y,n} &= 2.2\% \quad \forall n \\
\sigma_{y,n} &= 8.7\% \quad \forall n \\
\bar{r}(V^T(t)) &= 4.0\% \quad \forall n \\
\sigma(V^T(t)) &= 8.7\% \quad \forall n.
\end{align*}
\]

Yet, as we demonstrate in Figure 10, combining the stationary leverage ratio dynamics in equation (92) with the accounting identity in equation (91) generates dividend dynamics such that the term structures of (i) dividend volatility, (ii) excess returns on dividend strips, and (iii) return volatilities on dividend strips are all downward sloping.

Moreover, this simple model also generates long-horizon “excess volatility” in that stock return volatility \( \sigma_d = 13.4\% \) is significantly larger than long-horizon dividend volatility \( \sigma_T^{\infty}(t) = 8.7\% \) (which equals EBIT volatility). As is well understood (Campbell and Shiller (1988), Cochrane (1991, 2007)), “excess volatility” can be traced back to time-variation in discount rates or predictability in dividends. In our framework, we have both, in that leverage predicts both future dividends (according to equations (92) and (91), dividends are a function of leverage) and expected excess returns on equity. Expected excess returns on stocks are time-varying despite the fact that risk premia are constant, simply due to the time-variation in leverage.\(^\text{24}\)

\(^\text{24}\) The results from the full analysis of this model are available upon request.
C. Discussion

Even though in this section we focus on a different asset pricing framework than in Section III, with time-varying expected returns and no cash flow predictability, we find similar patterns when looking at the term structure of dividend strip return volatilities. With endogenous dividend dynamics derived from a similar mean-reverting process for aggregate leverage, we find that the short-term dividend claims are riskier than the long-term claims. As a result, they display higher volatility and expected returns than long-term claims. Taken together, these results suggest that time-varying (but stationary) leverage ratios are a first-order determinant of key properties of dividend dynamics and prices of the dividend strips.
V. Conclusion

Many leading asset pricing models, such as CC and BY, predict that the term structure of excess returns and volatilities of dividend strips are strongly upward sloping. However, the empirical findings of BBK suggest otherwise. We first show that, in contrast to the predictions of these leading models, empirical estimates for the VRs of dividends are decreasing with horizon. We then modify these leading models by retaining their pricing kernels, but replacing their dividend dynamics with processes that are consistent with this empirical fact. We show that this modification allows these models to retain their ability to match salient features of stock and bond returns while simultaneously generating dividend strips with decreasing term structures of expected returns and volatilities.

Further, we provide an economic mechanism that explains why dividend VRs should be decreasing with horizon. Specifically, we determine “endogenous” dividend dynamics by combining exogenously specified unlevered cash flow (i.e., EBIT) dynamics with a dynamic capital structure policy that generates stationary leverage ratios. This approach generates dividends that are riskier than EBIT in the short run, but cointegrated with EBIT in the long run. Intuitively, this is because, when a firm rebalances its debt levels over time to maintain a stationary leverage process, shareholders are being forced to divest (invest) when the firm does well (poorly). This interaction transfers risk from long-horizon dividends to short-horizon dividends, pushing downward the term structure of dividend strip volatilities.

By investigating several different economies, we demonstrate that this mechanism is robust and of first-order importance when it comes to pricing dividend strips. Moreover, the models can be calibrated to generate short-horizon dividend strips with low CAPM betas and high CAPM alphas, consistent with the empirical findings of BBK and BHKV.

Our modified dividend process also helps explain long-horizon “excess volatility” in that it generates models where stock returns are more volatile than long-horizon dividend volatility, even if the market prices of risk are constant. This prediction is more in line with observation, and eliminates a counterfactual prediction of the original BY model that stock return volatility should be significantly smaller than long-horizon dividend volatility.

Retaining the spirit of the endowment economies of CC and BY, we propose a joint model for EBIT and dividend payments without explicitly modeling firms’ investment decisions. However, we emphasize that, even if we had the correct pricing kernel and the correct investment and capital structure policies, we would still be left with the task of distinguishing between cash flows paid out as cash dividends (which are the relevant cash flows for determining the prices and returns of dividend strips) and those paid out as net equity repurchases. As shown in Table II, the two time series are extremely different, with short-horizon volatilities differing by a factor of almost two. This fact alone suggests that it would be far more complex to model dividend dynamics from first principles. As such, it seems more pragmatic to ensure that measurable properties
of dividend dynamics, such as (1) the downward-sloping term structure of dividend volatilities, and (2) the weak correlation between dividend growth and stock returns, be captured by any model of dividends used to price strips.

Initial submission: May 18, 2012; Final version received: October 31, 2014
Editor: Campbell Harvey

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher’s website:

**Appendix S1:** Internet Appendix.

**Table S.II.** Dividend Autocorrelation/Predictability.
Table S.III. Leverage and Time-Varying Risk Premia.
Figure S.1. Approximation errors.
Figure S.2. Impact of economic state.
Figure S.3. Leverage ratio and bondholders and stockholders payouts.
Figure S.4. Gross versus net leverage ratio.